

DOI: <https://doi.org/10.36602/jsba.2026.22.11>*Yasser-Jassim Transform–Variational Framework for Solving Selected Fractional Partial Differential Equations*Sameehah R. Alkaleeli^{1*}, Najiyah Amir²¹Department of Mathematics, Faculty of Arts and Sciences, Al-Marj, Benghazi University, Libya.²Department of Mathematics, Faculty of Sciences, Benghazi University, Libya*Corresponding author: sameehah.alkaleeli@uob.edu.ly

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Abstract: The primary purpose of this examination is to develop an innovative analytical technique for solving both linear and nonlinear fractional partial differential equations (FPDEs) by integrating the Variational Iteration Method (VIM) with the Yasser-Jassim Transform. This mixed approach utilizes the strengths of both methods to convert fractional derivatives into algebraic forms, simplifying the solution procedure. The Yasser-Jassim Transform excels at handling fractional derivatives, while the Variational Iteration Method provides a solid foundation for developing iterative solutions.

Some well-known equations serve to illustrate the validity and effectiveness of our integrated technique for treating FPDEs.

Keywords: Yasser-Jassim transform, inverse YJ-transform, Caputo derivative, differentiation property, time-fractional equation.

1. Introduction:

Numerous studies have focused on developing and applying advanced and efficient methods for solving fractional differential equations, such as the Adomian decomposition method [1], the homotopy perturbation method ([2], [3]), and the Variational Iteration Method ([4],[5]).

More recently, many researchers have used the Variational Iteration Method (VIM) combined with integral transforms, employing several such transformations, including the Laplace transform [6], Sumudu transform [7], Aboodh transform [8], and Shehu transform [9], among others.

In 2025, Mohammed T.Y. and Hassan K. J. [10] created another integral transform as a generalisation of the Laplace transform, which is called the Yasser-Jassim transform, and used it to solve ordinary differential equations and integral equations.

This work investigates the performance of the variation iteration method combined with the Yasser-Jassim transform applied to both linear and nonlinear fractional partial differential equations (PDEs). This kind of equations has the form:[11]

$${}^c D_\tau^\alpha \varphi(\chi, \tau) + L\varphi(\chi, \tau) + N\varphi(\chi, \tau) = \varphi(\chi, \tau), \quad n-1 < \alpha \leq n, \quad n = 1, 2, \dots \quad (1.1)$$

with initial conditions

$$\frac{\partial^{n-1} \varphi(\chi, 0)}{\partial \tau^{n-1}} = \eta_{n-1}(\chi), \quad (1.2)$$

where $D_\tau^\alpha = \frac{\partial^\alpha}{\partial \tau^\alpha}$ is the Caputo fractional derivative, and L, N symbolize the linear differential operator and the general nonlinear differential operator, respectively, $\varphi(\chi, \tau)$ is the source term.

2. Mathematical Background

To realize the success of the proposed approach, one must first grasp the essential components on which it is based: fractional calculus, the Yasser-Jassim transform, and the variational iterative method. This part begins with an overview of the essential concepts.

2.1 Fractional Calculus

Fractional calculus extends basic integer-order notions to include non-integer (fractional) orders. This concept is quite beneficial in mimicking systems that have memory behaviours or historical implications. Examples include heat diffusion in complex-structured materials and fluid flow in porous media. The definitions and remarks presented in this section are summarized from [3], [8], [12], and [13].

Definition 2.2

For a finite interval $Y = [a, b]$ ($-\infty < a < b < \infty$), and $\alpha \in \mathbb{R}$. The Riemann–Liouville fractional integral $I^\alpha \eta$ of order α is defined as [13]

$$(I^\alpha \eta)(\tau) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^\tau \frac{\eta(\chi)}{(\tau - \chi)^{1-\alpha}} d\chi, & \tau, \alpha > 0, \\ \eta(\tau), & \alpha = 0. \end{cases}$$

Here $\Gamma(\cdot)$ is the gamma function.

Theorem 2.1 [12] If $\eta(\tau) \in AC^n[a, b]$, then the Caputo fractional derivative $({}^c D_\tau^\alpha \eta)(\tau)$ exists almost everywhere on $[a, b]$. If $\alpha \notin \mathbb{N}$, $({}^c D_\tau^\alpha \eta)(\tau)$ is represented by

$$({}^c D_\tau^\alpha \eta)(\tau) = \frac{1}{\Gamma(n-\alpha)} \int_0^\tau \frac{\eta^{(n)}(\chi)}{(\tau-\chi)^{\alpha-n+1}} d\chi, \quad (2.1)$$

where $D = \frac{d}{d\tau}$, $\alpha \geq 0$ and $n = [\alpha] + 1$.

Remark2.1 In the sequel, we consider the time-fractional derivative in Caputo's sense. When $\alpha \in \mathbb{R}^+$, the time-fractional derivative is defined as

$$({}^c D_\tau^\alpha \eta)(\chi, \tau) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^\tau \frac{1}{(\tau-\chi)^{\alpha-n+1}} \frac{\partial^n \eta(\chi, \tau)}{\partial \tau^n} d\tau, & n-1 < \alpha \leq n, \\ \frac{\partial^n \eta(\chi, \tau)}{\partial \tau^n}, & \alpha = n, \end{cases} \quad (2.2)$$

where $n \in \mathbb{N}^*$.

Definition 2.3. The Mittag-Leffler function

$E_\alpha(\tau)$ is a generalization of the exponential function e^τ , and define by

$$E_\alpha(\tau) = \sum_{n=0}^{\infty} \frac{\tau^n}{\Gamma(n\alpha + 1)}, \quad \operatorname{Re}(\alpha) > 0, \tau \in \mathbb{C}. \quad (2.3)$$

2.2 The Yasser-Jassim Transform (YJT) [10]

The YJ-Transform is an integral transform invented by Mohammed T.Y. and Hassan K.J. in 2025; they used it to solve ODEs and IE, and it has proven successful. This transform has properties that make it ideal for fractional derivatives, simplifying the solution of complex equations.

Definition 2.4

Let $\eta(\tau)$ be an integrable function defined for $\tau \geq 0$. The YJ-transform of $\eta(\tau)$ is defined by the following formula

$$\begin{aligned} YJ(\eta(\tau)) &= K(a) = a \int_0^\infty e^{-\frac{1}{\sqrt{a}}\tau} \eta(\tau) d\tau \\ &= a \lim_{b \rightarrow \infty} \int_0^b e^{-\frac{1}{\sqrt{a}}\tau} \eta(\tau) d\tau; \quad a \neq 0. \end{aligned} \quad (2.4)$$

The inverse YJ-Transform is given as

$$YJ^{-1}[K(a)] = \eta(\tau).$$

2.2.1 YJ-Transforms of Some Basic Functions:

- $YJ(1) = a\sqrt{a}$.
- $YJ(e^{k\tau}) = \frac{a\sqrt{a}}{1-k\sqrt{a}}$.
- $YJ(\tau^n) = n! a^{\frac{n+1}{2}}, n = 0, 1, 2, \dots$.
- $YJ(\tau^\alpha) = a^{\frac{\alpha+1}{2}} \Gamma(\alpha+1), \alpha \geq 0$.

Existence, uniqueness, and other fundamental properties of the YJ-Transform are fully established in ([10], [14]).

Property2.2

Assume $K(a)$ and $G(a)$ are the YJ-Transform of $f(\tau)$ and $g(\tau)$, respectively. Then

- The YJ-Transform is a linear transformation, that is, for constants γ, β
 $YJ[\gamma f(\tau) + \beta g(\tau)] = \gamma K(a) + \beta G(a),$
- The YJ-transform of their convolution is given by

$$YJ[(f * g)(\tau)] = \frac{1}{a} K(a)G(a).$$

Theorem 2.3

Let $\eta(\tau)$ be an integrable function and $YJ(\eta(\tau)) = K(a)$, then for $n = 1, 2, 3, \dots$

$$\begin{aligned} YJ[\eta^{(n)}(\tau)] &= \frac{1}{a^{\frac{n}{2}}} YJ(\eta(\tau)) \\ &\quad - \sum_{k=0}^{n-1} a^{\frac{k-n+3}{2}} \eta^{(k)}(0), \end{aligned} \quad (2.5)$$

Theorem 2.4

If $K(a)$ is the YJ-transform of the function $\eta(\tau)$, then the YJ-Transform of

- the Riemann-Liouville fractional integral of the function $\eta(\tau)$ of order α , is given by
 $YJ(I^\alpha \eta(\tau)) = a^{\frac{\alpha}{2}} K(a), \alpha > 0,$
- the Caputo fractional derivative of $\eta(\tau)$ of order α , is given by

$$\begin{aligned} YJ({}^c D_\tau^\alpha \eta(\tau)) &= \frac{1}{a^{\frac{\alpha}{2}}} YJ[\eta(\tau)] \\ &\quad - \sum_{k=0}^{n-1} a^{\frac{k-\alpha+3}{2}} \eta^{(k)}(0), \\ n-1 &< \alpha \leq n, n \in \mathbb{N}. \end{aligned}$$

Proof

For this particular theorem, we used the approach of Ali Khalouta [15], which has been effective in previous evaluations of this type.

- Utilizing definition2.1, we have

$$\begin{aligned} (I^\alpha \eta)(\tau) &= \frac{1}{\Gamma(\alpha)} \int_0^\tau \frac{\eta(\chi)}{(\tau-\chi)^{1-\alpha}} d\chi, \\ &= \frac{1}{\Gamma(\alpha)} \int_0^\tau \eta(\chi)(\tau-\chi)^{\alpha-1} d\chi, \\ &= \frac{1}{\Gamma(\alpha)} \int_0^\tau (\tau-\chi)^{\alpha-1} \eta(\chi) d\chi, \\ &= \frac{1}{\Gamma(\alpha)} \tau^{\alpha-1} * \eta(\tau). \end{aligned}$$

Thus,

$$(I^\alpha \eta)(\tau) = \frac{1}{\Gamma(\alpha)} \tau^{\alpha-1} * \eta(\tau). \quad (2.6)$$

Operating the YJ-transform on both sides of Eq (2.6) and utilizing property 2.2(ii), we obtain

$$YJ(I^\alpha \eta(\tau)) = \frac{1}{\Gamma(\alpha)} \left(\frac{1}{a} YJ(\tau^{\alpha-1}) YJ(\eta(\tau)) \right).$$

We have,

$$YJ(\tau^{\alpha-1}) = a \frac{\alpha}{a^2} \Gamma(\alpha).$$

Therefore,

$$YJ(I^\alpha \eta(\tau)) = a \frac{\alpha}{a^2} K(a).$$

- ii. Let $f(\tau) = \eta^{(k)}(\tau)$, utilizing definition 2.2, we get

$$\begin{aligned} {}^c D^\alpha \eta(\tau) &= \frac{1}{\Gamma(n-\alpha)} \int_0^\tau \frac{\eta^{(n)}(\chi)}{(\tau-\chi)^{\alpha-n+1}} d\chi, \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^\tau (\tau-\chi)^{n-\alpha-1} \eta^{(n)}(\chi) d\chi, \\ &= \frac{1}{\Gamma(n-\alpha)} \int_0^\tau (\tau-\chi)^{n-\alpha-1} f(\chi) d\chi, \\ &= I^{n-\alpha} f(\tau). \end{aligned}$$

Thus

$${}^c D^\alpha \eta(\tau) = I^{n-\alpha} f(\tau). \quad (2.7)$$

Applying the YJ-transform to both sides of Eq. (2.7) and using Theorem 2.2, we obtain

$$\begin{aligned} YJ({}^c D^\alpha \eta(\tau)) &= YJ(I^{n-\alpha} f(\tau)) \\ &= a \frac{n-\alpha}{a^2} YJ(f(\tau)), \\ &= a \frac{n-\alpha}{a^2} YJ(\eta^{(k)}(\tau)), \\ &= a \frac{n-\alpha}{a^2} \left(\frac{1}{a^2} YJ(\eta(\tau)) - \sum_{k=0}^{n-1} a \frac{k-n+3}{a^2} \eta^{(k)}(0) \right), \\ &= \frac{1}{a^2} YJ(\eta(\tau)) - \sum_{k=0}^{n-1} a \frac{k-\alpha+3}{a^2} \eta^{(k)}(0). \end{aligned}$$

Proposition 2.2

The Yasser-Jassim transform of the function $\eta(\chi, \tau)$ with the Caputo fractional derivative of order α is given by

$$\begin{aligned} YJ[({}^c D_t^\alpha \eta)(\chi, \tau)] &= \frac{1}{a^2} YJ[\eta(\chi, \tau)] \\ &\quad - \sum_{k=0}^{n-1} a \frac{(k-\alpha+3)}{a^2} \frac{\partial^k \eta(\chi, 0)}{\partial \tau^k}, \\ n-1 &< \alpha \leq n, \\ n &= 1, 2, \dots \end{aligned} \quad (2.8)$$

2.3 Variational Iterative Method (VIM):

The variational iterative method is an efficient semi-numerical analytical approach for solving linear and nonlinear differential equations. It relies on constructing an iterative correction function that allows rapid convergence to an exact or approximate solution.

2.3.1 Corrective functional construction

To resolve a general nonlinear PDEq. (1.1), a correction function can be constructed as follows:

$$\begin{aligned} \vartheta_{n+1}(\chi, \tau) &= \vartheta_n(\chi, \tau) \\ &+ \int_0^\tau \gamma(\chi, \iota) [{}^c D_t^\alpha \vartheta_n(\chi, \iota) + L\vartheta_n(\chi, \iota) + N\vartheta_n(\chi, \iota) \\ &\quad - \varphi(\chi, \iota)] d\iota. \end{aligned}$$

The iterative formulation involves a Lagrange parameter γ , whose value is computed via variational theory. The subscript n represents the order of approximation in the solution sequence, where ϑ_n is a function according to the restricted covariance constraint, i.e., the covariance is zero.

3. Analysis of FVIYJM

Yasser-Jassim's fractional variational iterative transform (FVIYJM) technique combines the YJ-transformation with variational iterative techniques to provide a unified framework for solving FPDEs. To discuss the solution procedure of this method, we take a general nonlinear PDE of fractional order as in Eq. (1.1); we do the following basic steps:

- Utilizing the YJ-transform on both sides of (1.1), we get $YJ[{}^c D_t^\alpha \vartheta(\chi, \tau)] + YJ[L\vartheta(\chi, \tau)] + YJ[N\vartheta(\chi, \tau)] = YJ[\varphi(\chi, \tau)]$.

Then Proposition 2.2 enables us to write

$$\begin{aligned} YJ[\vartheta(\chi, \tau)] &= \sum_{k=0}^{n-1} a \frac{(k+3)}{a^2} \frac{\partial^k \vartheta(\chi, 0)}{\partial \tau^k} \\ &\quad + a \frac{\alpha}{a^2} [YJ[\varphi(\chi, \tau)]] \\ &\quad - a \frac{\alpha}{a^2} [YJ[L\vartheta(\chi, \tau)]] \\ &\quad + YJ[N\vartheta(\chi, \tau)]. \end{aligned} \quad (3.1)$$

- Operating the inverse YJ-transform on both sides of Eq. (3.1), we obtain $\vartheta(\chi, \tau) = G(\chi, \tau) - YJ^{-1} \left\{ a \frac{\alpha}{a^2} [YJ[L\vartheta(\chi, \tau)]] + YJ[N\vartheta(\chi, \tau)] \right\}$.

Where $G(\chi, \tau)$, represents the term arising from the source term and the prescribed initial conditions. Applying $\frac{\partial}{\partial \tau}$ on both sides of Eq.(3.2), we have

$$\frac{\partial \vartheta(\chi, \tau)}{\partial \tau} + \frac{\partial}{\partial \tau} \left(YJ^{-1} \left\{ a^{\frac{\alpha}{2}} [YJ[L\vartheta(\chi, \tau)] + YJ[N\vartheta(\chi, \tau)]] \right\} - \frac{\partial G(\chi, \tau)}{\partial \tau} \right) = 0. \quad (3.3)$$

According to the VIM [5], [16], we can construct a correct functional as follows

$$\vartheta_{n+1}(\chi, \tau) = \vartheta_n(\chi, \tau) - \int_0^\tau \left[\frac{\partial \vartheta_n(\chi, \iota)}{\partial \iota} + \frac{\partial}{\partial \iota} \left(YJ^{-1} \left\{ \left(a^{\frac{\alpha}{2}} \right) [YJ[L\vartheta_n(\chi, \iota)] + YJ[N\vartheta_n(\chi, \iota)]] \right\} - \frac{\partial G(\chi, \iota)}{\partial \iota} \right) \right] d\iota. \quad (3.4)$$

We have, $\vartheta(\chi, \tau) = \lim_{n \rightarrow \infty} \vartheta_n(\chi, \tau)$.

For numerical purposes, we can use the $(n+1)^{th}$ approximation, or this approach may give the exact solution if a closed-form one exists.

4. Application and analysis of results

In this section, the effectiveness of this approach is evaluated by testing it on three commonly encountered FPDEs, as shown below, and comparing the results with other PFDE solutions in terms of accuracy and efficiency.

4.1 Time-fractional Cauchy reaction–diffusion equation

Consider the equation as [12]

$$\begin{cases} {}^C D_t^\alpha \vartheta(\chi, \tau) = \vartheta_{\chi\chi}(\chi, \tau) - (4\chi^2 - 2\tau + 2)\vartheta(\chi, \tau), \\ \quad \quad \quad 0 < \alpha \leq 1, \\ \vartheta(\chi, 0) = e^{\chi^2}. \end{cases} \quad (4.1)$$

Utilizing the YJ-transform on both sides of (4.1) and using its differentiation property, we get

$$YJ[\vartheta(\chi, \tau)] = a^{\frac{3}{2}} e^{\chi^2} + \left(a^{\frac{\alpha}{2}} \right) \left[YJ \left[\vartheta_{\chi\chi}(\chi, \tau) - (4\chi^2 - 2\tau + 2)\vartheta(\chi, \tau) \right] \right]. \quad (4.2)$$

Taking the inverse YJ-transform of Eq. (4.2), we can find

$$\vartheta(\chi, \tau) = e^{\chi^2} + YJ^{-1} \left\{ \left(a^{\frac{\alpha}{2}} \right) \left[YJ \left[\vartheta_{\chi\chi}(\chi, \tau) - (4\chi^2 - 2\tau + 2)\vartheta(\chi, \tau) \right] \right] \right\}. \quad (4.3)$$

Operating $\frac{\partial}{\partial \tau}$ on both sides of Eq. (4.3), we have

$$\begin{aligned} & \frac{\partial \vartheta(\chi, \tau)}{\partial \tau} \\ & - \frac{\partial}{\partial \tau} \left(\left(a^{\frac{\alpha}{2}} \right) \left[YJ \left[\vartheta_{\chi\chi}(\chi, \tau) - (4\chi^2 - 2\tau + 2)\vartheta(\chi, \tau) \right] \right] \right) \\ & = 0. \end{aligned} \quad (4.4)$$

Using Eq. (3.4), we obtain

$$\vartheta_{n+1}(\chi, \tau) = \vartheta_n(\chi, \tau) - \int_0^\tau \left[\frac{\partial \vartheta_n(\chi, \iota)}{\partial \iota} - \frac{\partial}{\partial \iota} \left(YJ^{-1} \left\{ \left(a^{\frac{\alpha}{2}} \right) \left[YJ \left[\vartheta_{n,\chi\chi}(\chi, \iota) - (4\chi^2 - 2\iota + 2)\vartheta_n(\chi, \iota) \right] \right] \right\} \right) \right] d\iota. \quad (4.5)$$

Utilizing formula (4.5), the first iterations are given by

$$\begin{aligned} \vartheta_0(\chi, \tau) &= e^{\chi^2}, \\ \vartheta_1(\chi, \tau) &= e^{\chi^2} \left(1 + \frac{2\tau^{\alpha+1}}{\Gamma(\alpha+2)} \right), \\ \vartheta_2(\chi, \tau) &= e^{\chi^2} \left(1 + \frac{2\tau^{\alpha+1}}{\Gamma(\alpha+2)} \right. \\ & \quad \left. + \frac{4\Gamma(\alpha+3)\tau^{2\alpha+2}}{\Gamma(\alpha+2)\Gamma(2\alpha+3)} \right), \end{aligned}$$

$$\begin{aligned} \vartheta_3(\chi, \tau) &= e^{\chi^2} \left(1 + \frac{2\tau^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{4\Gamma(\alpha+3)\tau^{2\alpha+2}}{\Gamma(\alpha+2)\Gamma(2\alpha+3)} \right. \\ & \quad \left. + \frac{8\Gamma(\alpha+3)\Gamma(2\alpha+4)\tau^{3\alpha+3}}{\Gamma(\alpha+2)\Gamma(2\alpha+3)\Gamma(3\alpha+4)} \right), \\ & \vdots \end{aligned}$$

Hence, the solution of Eq.(4.1), is given as

$$\begin{aligned} \vartheta(\chi, \tau) &= e^{\chi^2} \left(1 + \frac{2\tau^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{4\Gamma(\alpha+3)\tau^{2\alpha+2}}{\Gamma(\alpha+2)\Gamma(2\alpha+3)} + \right. \\ & \quad \left. \frac{8\Gamma(\alpha+3)\Gamma(2\alpha+4)\tau^{3\alpha+3}}{\Gamma(\alpha+2)\Gamma(2\alpha+3)\Gamma(3\alpha+4)} + \dots \right), \\ &= e^{\chi^2} \left(1 + \frac{2\tau^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{4(\alpha+2)\tau^{2\alpha+2}}{\Gamma(2\alpha+3)} + \right. \\ & \quad \left. \frac{8(\alpha+2)(2\alpha+3)\tau^{3\alpha+3}}{\Gamma(3\alpha+4)} + \dots \right), \quad 0 < \alpha \leq 1. \end{aligned}$$

The obtained solution is associated with the results reported by Kumar S [12]. Furthermore, by setting $\alpha = 1$, the exact solution of Eq. (4.1) is given by $\vartheta(\chi, \tau) = e^{\chi^2 + \tau^2}$.

Table1: Comparison of the Exact Sol. and app. sol $\vartheta(\chi, \tau)$ Eq. (4.1) for $\alpha = 1, 0.55, n = 4$.

χ	τ	Exact Sol.	App. Sol $\alpha = 1$	App. Sol $\alpha = 0.55$	$E_{abs}(\alpha = 1)$	$E_{abs}(\alpha = 0.55)$
0.25	0.25	1.13314845	1.13314777	1.26851158	$6.85336109 \times 10^{-7}$	0.13536313
	0.50	1.36683794	1.36665565	1.83118875	1.8229×10^{-4}	0.46435081
	0.75	1.86824596	1.86325513	3.05695091	4.99082×10^{-3}	1.18870496
	1	2.89359594	2.83865189	5.53640224	5.494405×10^{-2}	2.6428063
0.50	0.25	1.36683794	1.36683711	1.53011704	$8.26673145 \times 10^{-7}$	0.1632791
	0.50	1.64872127	1.64850138	2.20883527	2.1989×10^{-4}	0.560114
	0.75	2.25353479	2.2475147	3.68738666	6.02008×10^{-3}	1.43385188
	1	3.49034296	3.42406778	6.67817586	6.627518×10^{-2}	3.1878329
0.75	0.25	1.86824596	1.86824483	2.09142203	$1.12992822 \times 10^{-6}$	0.22317607
	0.50	2.25353479	2.25323423	3.01911985	3.0055×10^{-4}	0.76558506
	0.75	3.08021685	3.07198837	5.04006000	8.22848×10^{-3}	1.95984315
	1	4.77073318	4.68014575	9.12798414	9.058743×10^{-2}	4.35725096

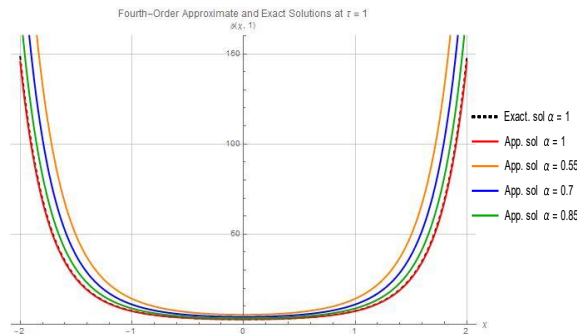


Figure 1: 2D Surface behavior of 4th-order approximate solutions of Eq. (4.1) over $-2 \leq \chi \leq 2$ at $\tau = 1$ for different values of α and the exact solution.

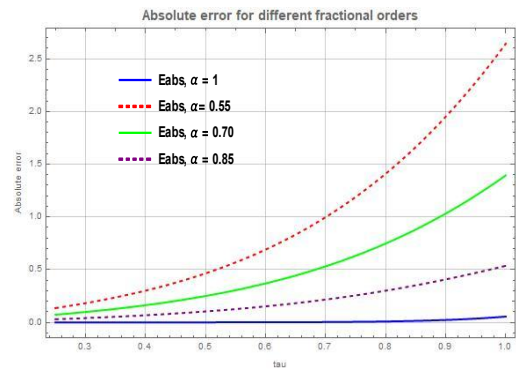


Figure 2: Absolute error for different values of α , with $\tau = 0.25, 0.50, 0.75, 1, \chi = 0.25, 0.50, 0.75$.

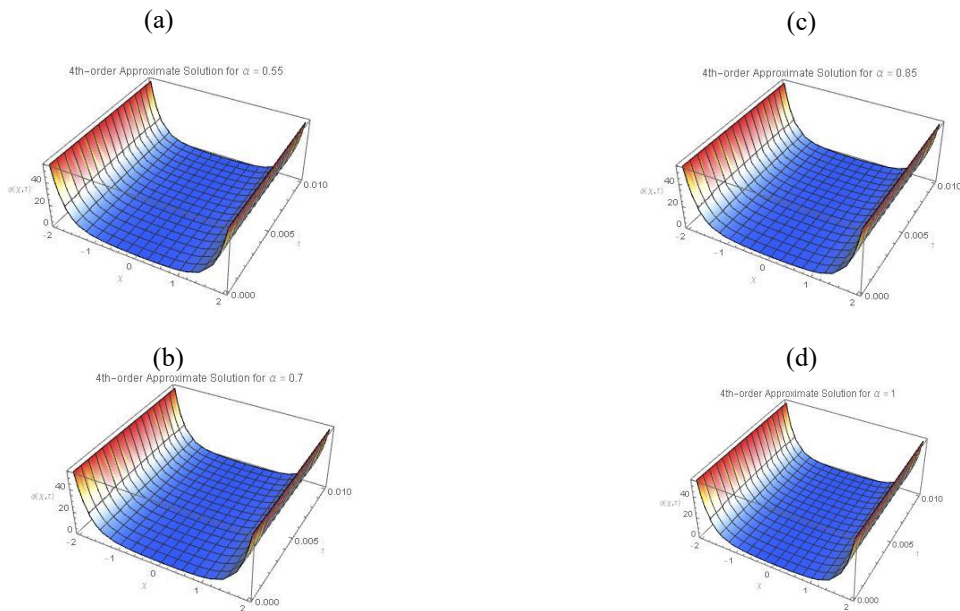


Figure 3: 3D comparison of the fourth-order approximate solution $\vartheta(\chi, \tau)$ of Eq. (4.1) for different values of α : (a) $\alpha = 0.55$, (b) $\alpha = 0.7$, (c) $\alpha = 0.85$, and (d) $\alpha = 1$.

4.2 Nonlinear time-fractional gas dynamics equation [7]

The equation is given by

$$\begin{cases} {}^c D_t^\alpha \vartheta(\chi, \tau) + \frac{1}{2} \vartheta_\chi^2 - \vartheta(1 - \vartheta) = 0, & \tau > 0, 0 < \alpha \leq 1, \\ \vartheta(\chi, 0) = e^{-\chi}. \end{cases} \quad (4.6)$$

Utilizing the proposed method on Eq. (4.6), and using the obtained formula, the first iterations are given as

$$\begin{aligned} \vartheta_0(\chi, \tau) &= e^{-\chi}, \\ \vartheta_1(\chi, \tau) &= e^{-\chi} \left(1 + \frac{\tau^\alpha}{\Gamma(\alpha + 1)} \right), \end{aligned}$$

$$\begin{aligned} \vartheta_2(\chi, \tau) &= e^{-\chi} \left(1 + \frac{\tau^\alpha}{\Gamma(\alpha + 1)} + \frac{\tau^{2\alpha}}{\Gamma(2\alpha + 1)} \right), \\ \vartheta_3(\chi, \tau) &= e^{-\chi} \left(1 + \frac{\tau^\alpha}{\Gamma(\alpha + 1)} + \frac{\tau^{2\alpha}}{\Gamma(2\alpha + 1)} + \frac{\tau^{3\alpha}}{\Gamma(3\alpha + 1)} \right), \\ &\vdots \\ \vartheta_n(\chi, \tau) &= e^{-\chi} \sum_{k=0}^n \frac{\tau^{k\alpha}}{\Gamma(k\alpha + 1)}. \end{aligned}$$

Then, the approximate solution of Eq. (4.6) is, $\vartheta(\chi, \tau) = \lim_{n \rightarrow \infty} \vartheta_n(\chi, \tau)$,

$$= \lim_{n \rightarrow \infty} e^{-\chi} \sum_{k=0}^n \frac{\tau^{k\alpha}}{\Gamma(k\alpha + 1)},$$

$$= e^{-\chi} E_{\alpha}(\tau^{\alpha}), \quad 0 < \alpha \leq 1.$$

The obtained solution is associated with the results reported by Singh, J., Kumar, D., and Kılıçman, A. [17], as well as Abedl-Rady et al. [7]. Furthermore, by setting $\alpha = 1$, the exact solution of Eq(4.6) is given by $\vartheta(\chi, \tau) = e^{\tau-\chi}$.

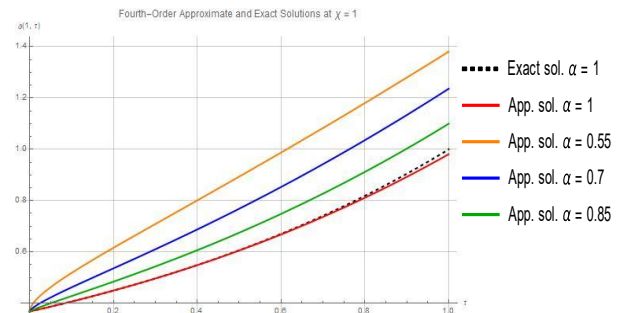


Figure 4: 2D Surface behavior of the 4th-order approximate solution of Eq. (4.6) when $\chi=1$ for different values of α and the exact solution.

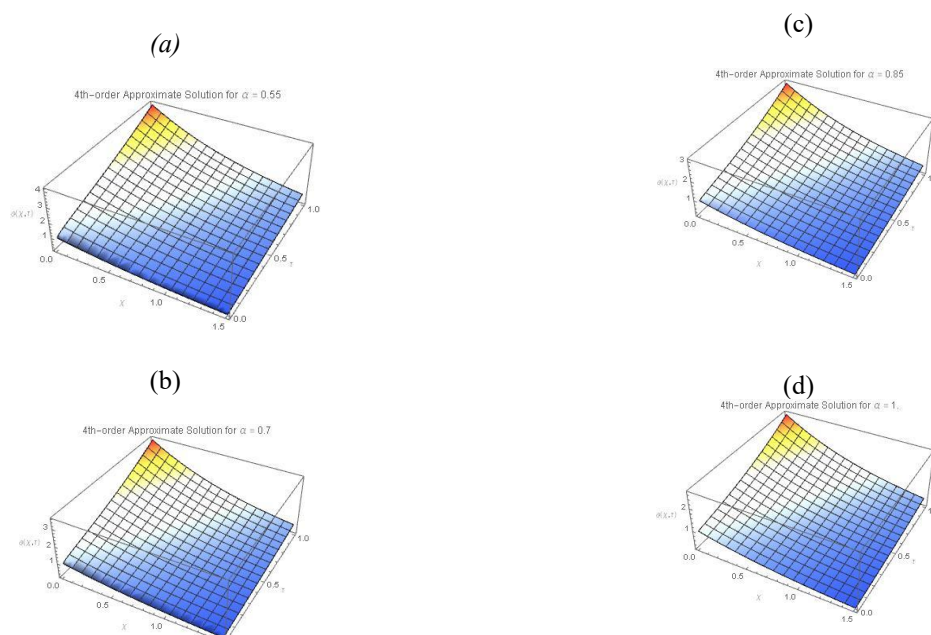


Figure 5: 3D comparison of the fourth-order approximate solution $\vartheta(\chi, \tau)$ of Eq. (4.6) for different values of α : (a) $\alpha = 0.55$, (b) $\alpha = 0.7$, (c) $\alpha = 0.85$, and (d) $\alpha = 1$.

4.3 The time-fractional Fisher's equation

(TFFE): The TFFE equation is defined as

$$\begin{cases} {}^c D_{\tau}^{\alpha} \vartheta(\chi, \tau) = \vartheta_{\chi\chi} + 6\vartheta(1 - \vartheta), & \tau > 0, \\ 0 < \alpha \leq 1, \\ \vartheta(\chi, 0) = \frac{1}{(1+e^{\chi})^2}. \end{cases} \quad (4.7)$$

Utilizing the proposed method on Eq. (4.7), and then using the obtained formula, the first iterations are given as

$$\vartheta_0(\chi, \tau) = \frac{1}{(1+e^{\chi})^2},$$

$$\begin{aligned} \vartheta_1(\chi, \tau) &= \frac{1}{(1+e^{\chi})^2} \\ &+ \left(\frac{6e^{2\chi}}{(1+e^{\chi})^4} - \frac{2e^{\chi}}{(1+e^{\chi})^3} \right. \\ &+ \frac{6}{(1+e^{\chi})^2} \\ &\left. - \frac{6}{(1+e^{\chi})^4} \right) \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}, \\ &= \frac{1}{(1+e^{\chi})^2} + \frac{10e^{\chi}}{(1+e^{\chi})^3} \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}, \end{aligned}$$

$$\begin{aligned} \vartheta_2(\chi, \tau) &= \frac{1}{(1+e^\chi)^2} + \frac{10e^\chi}{\tau^\alpha} \\ &+ \frac{(1+e^\chi)^3 \Gamma(\alpha+1)}{50e^\chi(2e^\chi-1)} \frac{\tau^{2\alpha}}{\tau^{3\alpha} \Gamma(2\alpha+1)} \\ &- \frac{(1+e^\chi)^4}{600e^{2\chi}} \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}, \\ &\vdots \\ \vartheta_n(\chi, \tau) &= \frac{1}{(1+e^\chi)^2} + \frac{10e^\chi}{(1+e^\chi)^3} \frac{\tau^\alpha}{\Gamma(\alpha+1)} + \\ &\frac{50e^\chi(2e^\chi-1)}{\tau^{2\alpha}} - \frac{(1+e^\chi)^4}{600e^{2\chi}} \frac{\Gamma(2\alpha+1)}{\tau^{3\alpha} \Gamma(2\alpha+1)} \\ &- \frac{(1+e^\chi)^6}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} + \dots \end{aligned}$$

Eq. (4.7) becomes,

$$\vartheta_\tau(\chi, \tau) = \vartheta_{\chi\chi} + 6\vartheta(1-\vartheta), \quad \tau > 0,$$

when $\alpha = 1$. Which has an exact solution,

$$\vartheta(\chi, \tau) = \frac{1}{(1+e^{\chi-5\tau})^2}.$$

As shown by Al-deiakkeh, R., et al [18].

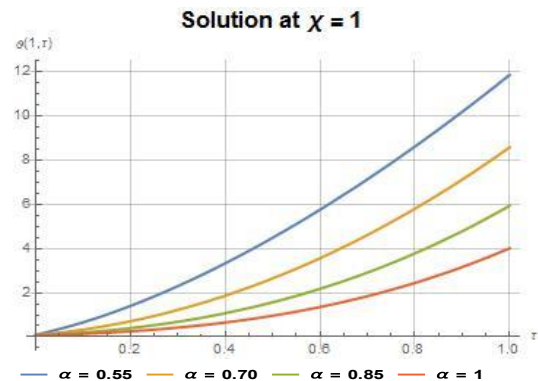


Figure 6: 2D Surface behavior of the 4th-order approximate solution of Eq. (4.7) when $\chi=1$ for different values of α and the exact solution.

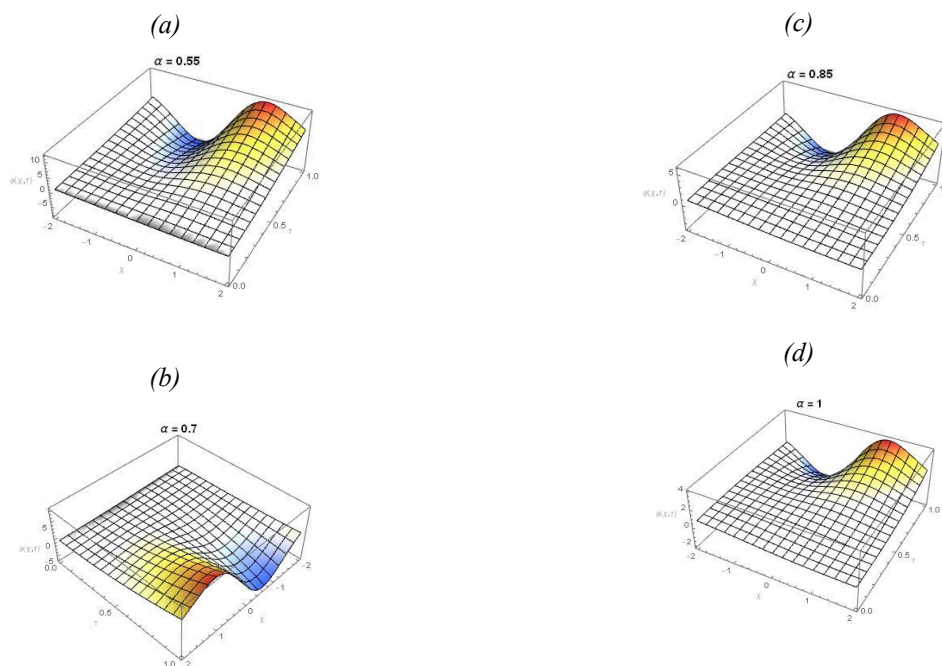


Figure 7: 3D comparison of the third-order approximate solution $\vartheta(\chi, \tau)$ of Eq. (4.9) for different values of α (a) $\alpha = 0.55$, (b) $\alpha = 0.7$, (c) $\alpha = 0.85$, and (d) $\alpha = 1$.

5. Results and Discussion

Three numerical examples were provided to demonstrate the suggested FVIYJTM approach for solving FPDEs.

For the first example(4.1),

- The exact solution was compared with the four-term FVIYJTM for different values of χ , τ , and α .

- Table 1 compares the exact solution with the four-term approximate solution $\vartheta(\chi, \tau)$ of Eq. (4.1) for $\alpha = 1$ and $\alpha = 0.55$. It also presents the absolute error for different values of χ and τ .
- The results show that $\alpha = 1$ yields a more accurate approximation, whereas $\alpha = 0.55$ causes larger errors, especially for large values of τ .
- Figures 1, 2, and 3 extend this comparison to all considered fractional orders $\alpha = 1, 0.55, 0.70$, and 0.85 . They show that the error increases as the fractional order decreases, with the largest error occurring for $\alpha = 0.55$.

Thus, the table provides a numerical comparison, while the figures illustrate the behaviour of the solutions and errors across all fractional orders of Example 4.1, proving that as the fractional order α approaches 1, the approximate solution becomes increasingly close to the exact solution.

The results of the second and third examples were reported more succinctly; they showed similar agreement with the previous studies ([7], [17], and [18]) and confirmed the applicability of the proposed method to FPDEs.

6. CONCLUSION

This work proposes an innovative and efficient analytical approach for solving linear and nonlinear fractional partial differential equations. The methodology, called FVIYJTM, combines the power of the Yasser-Jassim transform with the advantages of the iterative variational approach. The application examples presented in the article demonstrate the efficiency and relevance of this method. FVIYJTM significantly reduces computational complexity. FVIYJTM is effective for researchers and engineers working on complex processes using fractional modelling. It is easy to use and can handle nonlinearities, making it an excellent solution for future

applications across all scientific and technical fields.

7. References

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