Calculating The Rotational Energy Levels for Ground State Bands of Dy , Gd and Er Even-Even Isotopes

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Abstract: The aim of this work is to calculate the rotational energy levels and energy of transition of 156-162 Dy, Gd and Er even-even Isotopes by using the Nuclear Softness Model (NS3) and approximated Model of the NS2 (our work). The calculated values for both models were obtained by MATLAB program and compared with the experimental data, it was a good agreement.

Keywords: The rotational energy levels, Ground State band (GSB), The Nuclear Softness Model, Dy, Gd and Er isotopes.

Introduction

There are many models for studying the energy levels of the ground state bands of even – even deformed nuclei.

Al-Malki (2005) [1] studied the energy levels for g-s bands of some deformed nuclei at A~150 by using several nuclear models.

While Kassim and et al (2020) [2] studied the energy levels of ^{158}Er, ^{160}Yb and ^{162}Hf isotopes by using the Boher model (BM), the interacting boson model (IBM) and the interacting vector boson model (IVBM). The models shows good results for these isotopes.

Najem andAl-Jubbori (2021) used the interacting boson model (IBM-1) to calculate the energy levels for $^{158-170}$Dy even-even isotopes for the three limits (U(5), O(6) and SU(3)). The results compared with the experimental data and shows a good agreement.

In the present work, the aim purpose is to study the energy levels of the ground state rotational of some isotopes for Dy (Dysprosium), Gd (Gadolinium) and Er (Erbium) even-even by using the Nuclear Softness Model-3 (NS3) and correction model.

The Nuclear Softness Model

According to Boher - Motteson for calculating the rotational energy levels of the ground – state bands by using classical mechanics in even-even nuclei are given by [4]

$$E(I) = \frac{\hbar^2}{2\,\phi}\ [I\ (I+1)] \qquad (1)$$

Where I is the angular momentum or spin ($I = 0^+, 2^+, 4^+, 6^+, \ldots\ldots.$), $\hbar = \frac{h}{2\,\pi}$, $h = 6.62607015 \times 10^{-34}\ Kg\ m^2\ /s$ and ϕ is the moment of inertia for g- state.

The variation of the moment of inertia depends as a function of angular momentum (I) [5]. By using the Taylor series expansion of $\phi(I)$ [6], Eq. (1) becomes

$$E(I) = \frac{\hbar^2}{2}\left[\frac{1}{\phi_0} - \mathrm{I}\left(\frac{1}{\phi_I^2}\frac{\partial\phi_I}{\partial \mathrm{I}}\right)_0 + \frac{\mathrm{I}^2}{2!}\left(\frac{2}{\phi_I^3}\left(\frac{\partial\phi_I}{\partial \mathrm{I}}\right)^2 + -\frac{1}{\phi_I^2}\frac{\partial^2\phi_I}{\partial I^2}\right)_{I=0} + \cdots\right] I(I+1) \qquad (2)$$

Where ϕ_0 for I=0.

Based on the nuclear "softness" [7] that is described by the next equation

$$\sigma_n = \frac{1}{\phi_0}\frac{1}{n!}\frac{\sigma^n\phi(I)}{\sigma I^n}\bigg|_{I=0}, n = 1,2,3,\ldots \qquad (3)$$

then , we can write Eq. (2) by using Eq. (3), to becomes

$$E(I) = \frac{\hbar^2}{2\,\phi_0} I(I+1)\frac{1}{1+\sigma_1 I}\left[1 - \frac{\sigma_2\, I^2}{1+\sigma_1 I+\sigma_2\, I^2} - \frac{\sigma_3 I^3}{1+\sigma_1+\sigma_3 I^3} + \frac{\sigma_2\sigma_3 I^5}{1+\sigma_1 I+\sigma_2\sigma_3 I^5} + \cdots\right] \quad (4)$$

From Eq. (4), keeping the first order of the nuclear softness, and putting others equal to zero ($\sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = \cdots = 0$) [5], then we get

$$E(I) = \frac{\hbar^2}{2\,\phi_0}\,\frac{I(I+1)}{1+\sigma_1 I} \quad (5)$$

or

$$E(I) = \frac{A\ I\,(I+1)}{1+\sigma_1 I} \quad (6)$$

where $A = \frac{\hbar^2}{2\,\phi_0}$, so it has two parameters in Eq. (6) ($A\ and\ \sigma_1$), this equation is called the Nuclear Softness Two (NS2).

While keeping the first and second order ($\sigma_1\ and\ \sigma_2$) in Eq. (4), can increase the parameters to become three ($A, \sigma_1 and\ \sigma_2$), then Eq. (4) is given by

$$E(I) = \frac{A\ I(I+1)}{1+\sigma_1 I+\sigma_2\, I^2} \quad (7)$$

where Eq. (7) is the Nuclear Softness three (NS3). This equation is the one of the nuclear model that get better argument with experimental data [5].

In this paper, we add a new term to Eq. (6) to be

$$E(I) = \frac{A\ I\,(I+1)}{1+\sigma_1 I} - BI^2(I+1)^2 \quad (8)$$

where B is the parameter describe the rotational- vibration that related with γ and β [8].

We use Eqs (7) and (8) to calculate the energy of the rotational levels for the ground state bands (GSB) in 156-162 Dy, Gd and Er isotopes. The results were compared with the measured values for these even-even isotopes.

In addition , from equation (1), we get the energy of transition from state (I) to the next lower state (I-2), it given by

$$E(I) - E(I-2) = \frac{\hbar^2}{2\,\phi_0}\{I\ (I+1) - (I-2)[(I-2)+1]\} = \frac{\hbar^2}{2\,\phi_0}(4I-2) \quad (9)$$

where

$$E(I) - E(I-2) = E_\gamma(I) \quad (10)$$

$E_\gamma(I)$ is also called the gamma energies [9].

Results and Discussions

To get a calculation of the rotational energy levels of the ground- state bands of Dy, Gd and Er isotopes using NS3 and correction model (our work). Eq. (7) and Eq. (8) have been written in MATLAB programme to get the best fitted by using experimental data for ^{156}Gd, ^{156}Dy and ^{156}Er [10], [11] for ^{158}Gd, ^{158}Dy and ^{158}Er also other data [12], [13] and [14] for ^{160}Gd and ^{160}Dy , ^{160}Er and ^{162}Gd, ^{162}Dy and ^{162}Er respectively to find the parameters, and use it to calculate the energy levels E(I) for the both models. The energy levels that calculated of all nuclei isotopes, then use it to calculate the energy of a transition (E_γ) between $E(I)$ and $E(I-2)$ levels.

Gadolinium ($^{156-162}_{64}$Gd) has 64 protons with number of neutrons for these isotopes (that used in this paper) are N=92, 94, 96 and 98 respectively. While Dysprosium ($^{156-162}_{66}$Dy) has 66 with number of neutrons for N= 90, 92, 94 and 96, whereas Erbium ($^{156-162}_{68}$Er) has 68 protons with 88, 90, 92 and 94 neutrons.

Table 1 shows the parameters of each model were determined by fitting their equations with the measured energy values. The parameters of NS3 model are A (in keV), σ_1 (no unit) and σ_2 (no unit), while the correction model (our work Eq. 8) has these parameters A, σ_1 and B with units (keV), (no unit) and (keV) respectively.

These obtained parameters are used to calculate the energy levels of g-s bands in 156-162 Dy, Gd and Er even-even isotopes for NS3 model and correction model Eq. (8) (our work) that gives in Table 2 with Percentage Error (% error), it is written as

$$\%\,Error = \frac{E_{exp} - E_{cal}}{E_{exp}} \times 100 \quad (11)$$

From Table 2, in general, we can see that the calculated values of the energy levels for about most of all the isotopes of the nuclei are close to the experimental values, while the highest percentage error ranged (7.69 and 10.71 %) at $I^{+}=28^{+}$ and $I^{+}=30^{+}$ respectively for ${}^{158}_{66}Dy_{92}$ by NS3 model, whilst it was about 6 % (at $I^{+}=22^{+}$) and 7.86 % ($I^{+}=2^{+}$) in ${}^{158}_{68}Er_{90}$ for NS3 and our work respectively. However in ${}^{160}_{68}Er_{92}$ at $I^{+}=28^{+}$ and $I^{+}=30^{+}$ was 7.06 % and 10.11% respectively for NS3 model.

Fig.1 shows the comparison between the experimental data and calculated energy levels of g-state for NS3 model and our work of ${}^{156-162}_{64}$Gd isotopes with neutron number N=92-98, we can see that the calculated values are quite agree with the measured values.

In the case of ${}^{156-162}_{66}$Dy with neutron number N=90-96, the relationship between the energy levels for experimental and calculations as a function of angular momentum (I) that gives in Fig. 2. In this figure, we noticed that the every calculations for NS3 are slightly large with measured data at high spin while the correction model (our work) calculations are very similar with experimental data.

Fig. 3 shows the correction model (our work) calculations of the energy levels for ${}^{156-162}_{68}$Er isotopes with N=88-94 calculations are better overall than the NS3 and the experimental data, except for the calculations by NS3 model for ${}^{162}_{68}Er_{94}$, it is very quiet agreed with experimental data. While the NS3 calculations of energy levels for ${}^{156}_{68}Er_{88}$, ${}^{158}_{68}Er_{90}$ and ${}^{160}_{68}Er_{92}$ are large at high spin ($I^{+}=28^{+}$ and 30+).

Table 2 shows the energy of transition as a function of angular momentum (I) for g-s bands for the experimental data, NS3 model and our work calculations in even-even ${}^{156-162}$Gd , ${}^{156-162}$Dy and ${}^{156-162}$Er isotopes.

Conclusion

In present work we have focused on calculated the rotational energy levels of the g-s bands in even-even isotopes for Dy, Gd and Er nuclei by using NS3 model and our work. The calculations by our work (approximated Model of the NS2) give better agreement with measured data than the NS3 calculations. The results that show in this work have excellent agreement with experimentally data for the ground state bands. The energy of transition between two levels ($E_{\gamma}(I)$) that calculated in this paper, we can use it for studying some nuclear properties such as the back bending phenomena for these isotopes.

In the future, other nuclear models can be studied to study the nuclear properties of these or others nuclei.

Table 1 The NS3 model and correction model (our work) parametres of g-s bands for Gd, Dy and Er isotopes

Nucleus	NS3 Model			Our work (Eq.(8))		
	A (keV)	σ_1	σ_2	A (keV)	σ_1	B (keV)
${}^{156}_{64}Gd_{92}$	15.6775	0.0207	0.00022917	15.8	0.02383	-0.0007001
${}^{158}_{64}Gd_{94}$	13.4127	0.0049	0.00043699	13.5	0.006896	0.00331
${}^{160}_{64}Gd_{96}$	12.6573	0.0034	0.00035407	12.84	0.006498	0.002091
${}^{162}_{64}Gd_{98}$	12.0341	0.0031	0.00037658	12.19	0.006021	0.002271
${}^{156}_{66}Dy_{90}$	26.0520	0.0727	-0.00061266	26.1	0.06664	-0.0008881
${}^{158}_{66}Dy_{92}$	17.0835	0.0166	0.00070350	18.48	0.03868	-0.0003827
${}^{160}_{66}Dy_{94}$	14.6773	0.0058	0.00072917	15.7	0.0236	0.00085
${}^{162}_{66}Dy_{96}$	13.5604	0.0032	0.00052956	14.13	0.01189	0.001225
${}^{156}_{68}Er_{88}$	90.2623	0.3005	-0.0012	99.45	0.33	-0.001606
${}^{158}_{68}Er_{90}$	38.8947	0.1185	-0.0011	34.22	0.07978	0.001055
${}^{160}_{68}Er_{92}$	22.3387	0.0330	0.00068397	23.84	0.05588	-0.0005851
${}^{162}_{68}Er_{94}$	17.1621	0.0045	0.0013	19.01	0.0264	0.00444

Table 2 The experimenal and calculated energy levels of g-s bands of even-even nuclei

Nuclei	I^{π}	E_{exp} (keV)	E_{NS3}(keV)	(% error)	$E_{our\ work}$(keV)	(% error)
$^{156}_{64}Gd_{92}$	2^+	88.970	90.3	-1.50	90.46	-1.68
	4^+	288.187	288.6	-0.14	288.22	-0.01
	6^+	584.715	581.5	0.55	579.36	0.92
	8^+	965.134	956.6	0.88	951.83	1.38
	10^+	1416.078	1402.5	0.96	1395.07	1.48
	12^+	1924.49	1909.1	0.80	1899.66	1.29
	14^+	2475.82	2467.4	0.34	2457.08	0.76
	16^+	3059.5	3069.1	-0.31	3059.50	0
	18^+	3673.3	3707.1	-0.92	3699.62	-0.72
	20^+	4325.9	4374.8	-1.13	4370.56	-1.03
	22^+	5026.0	5066.7	-0.81	5065.71	-0.79
	24^+	5778.7	5777.5	0.02	5778.7	0
	26^+	6582.6	6503.0	1.21	6503.30	1.21
$^{158}_{64}Gd_{94}$	2^+	79.5143	79.6	-0.11	79.80	-0.36
	4^+	261.458	261.3	0.06	261.50	-0.02
	6^+	539.022	538.9	0.02	538.77	0.05
	8^+	904.12	904.7	-0.06	904.26	-0.02
	10^+	1349.5	1349.7	-0.02	1349.50	0
	12^+	1865	1864.5	0.03	1864.99	0
$^{160}_{64}Gd_{96}$	2^+	75.263	75.3	-0.05	75.96	-0.93
	4^+	245.502	248.4	0.04	249.41	-0.37
	6^+	514.81	514.5	0.06	515.26	-0.09
	8^+	868.6	868.0	0.07	867.79	0.09
	10^+	1301.3	1301.8	-0.04	1300.67	0.05
	12^+	1806.9	1808.4	-0.08	1806.91	0.0006
	14^+	2377.9	2379.4	-0.06	2378.87	-0.04
	16^+	3008.7	3006.3	0.08	3008.26	0.02
$^{162}_{64}Gd_{98}$	2^+	71.6	71.6	0	72.20	-0.84
	4^+	236.4	236.3	0.04	237.19	-0.33
	6^+	490.0	489.6	0.08	490.19	-0.04
	8^+	826.2	825.9	0.04	825.68	0.06
	10^+	1237.9	1238.4	-0.04	1237.44	0.04
	12^+	1718.6	1719.6	-0.06	1718.47	0.008
	14^+	2260.2	2261.3	-0.05	2261.06	-0.04
	16^+	2857.1	2855.3	0.06	2856.73	0.01
$^{156}_{66}Dy_{90}$	2^+	137.77	136.8	0.70	138.23	-0.33
	4^+	404.19	406.8	-0.65	412.53	-2.06
	6^+	770.44	773.9	-0.45	784.7231	-1.85
	8^+	1215.61	1216.4	-0.07	1230.4	-1.22
	10^+	1725.02	1720.8	0.25	1733.8	-0.51
	12^+	2285.88	2278.4	0.33	2284.2	0.07
	14^+	2887.82	2883.7	0.14	2875.0	0.44
	16^+	3523.3	3532.9	-0.27	3501.8	0.61
	18^+	4178.1	4222.5	-1.06	4162.5	0.37
	20^+	4859.0	4953.4	-1.94	4856.1	0.06
	22^+	5573.0	5724.3	-2.72	5583.1	-0.18
	24^+	6328.7	6535.0	-3.26	6344.8	-0.25
	26^+	7130.3	7386.2	-3.59	7143.2	-0.18
	28^+	7978.5	8278.7	-3.76	7981.1	-0.03
	30^+	8875.9	9213.7	-3.81	8861.9	-0.16

Nuclei	I^{π}	E_{exp} (keV)	E_{NS3}(keV)	(% error)	$E_{our\ work}$(keV)	(% error)
$^{158}_{66}Dy_{92}$	2^+	98.918	98.9	0.02	102.939	-4.07
	4^+	317.139	317.0	0.04	320.25	-0.98
	6^+	637.712	637.7	0.002	630.67	1.10
	8^+	1043.88	1044.2	-0.03	1018.16	2.46
	10^+	1520.0	1519.8	0.01	1470.51	3.26
	12^+	2048.8	2048.6	0.0098	1978.35	3.44
	14^+	2612.2	2617.4	-0.20	2534.47	2.98
	16^+	3190.3	3213.5	-0.73	3133.36	1.79
	18^+	3781.3	3826.3	-0.19	3770.84	0.28
	20^+	4407.1	4470.0	-1.43	4443.79	-0.83
	22^+	5085.2	5086.3	-0.02	5149.99	-1.27
	24^+	5819.9	5684.2	2.33	5887.95	-1.17
	26^+	6612.5	6290.2	4.87	6656.8	-0.67
	28^+	7456.2	6882.8	7.69	7456.2	0
	30^+	8354.2	7459.3	10.71	8286.28	0.81
$^{160}_{66}Dy_{94}$	2^+	86.7877	86.8	-0.01	89.93	-3.62
	4^+	283.8219	283.7	0.04	286.61	-0.98
	6^+	581.066	581.1	-0.006	576.18	0.84
	8^+	966.83	697.0	-0.02	946.58	2.10
	10^+	1427.89	1428.0	-0.008	1387.14	2.85
	12^+	1950.17	1950.0	-0.009	1888.23	3.18
	14^+	2513.36	2517.9	-0.2	2441.053	2.88
	16^+	3089.4	3120.2	-1.0	3037.43	1.68
	18^+	3669.65	3744.2	-2.03	3669.65	0
	20^+	4278.39	4379.2	-2.36	4330.34	-1.21
	22^+	4935.60	5016.3	-1.64	5012.36	-1.56
	24^+	5647.30	5648.0	-0.01	5708.71	-1.09
	26^+	6412.5	6268.4	2.25	6412.5	0
	28^+	7230.3	6872.8	4.95	7116.84	1.57
$^{162}_{66}Dy_{96}$	2^+	80.661	80.7	-0.05	82.74	-2.58
	4^+	265.664	265.6	0.02	269.19	-1.33
	6^+	548.520	548.6	-0.02	551.60	-0.56
	8^+	921.28	921.5	-0.02	922.34	-0.12
	10^+	1375.08	1374.9	0.01	1373.86	0.09
	12^+	1901.10	1897.8	0.17	1898.61	0.13
	14^+	2491.65	2479.3	0.50	2489.00	0.11
	16^+	3138.55	3108.0	0.97	3137.39	0.04
	18^+	3830.93	3773.0	1.51	3835.97	-0.13
	20^+	4577.73	4464.1	2.47	4576.83	0.01
	22^+	5352.0	5171.9	3.37	5351.84	0.003
	24^+	6153.2	5888.0	4.31	6152.66	0.009
$^{156}_{68}Er_{88}$	2^+	344.53	339.3	1.52	359.52	-4.35
	4^+	797.39	826.9	-3.70	858.01	-7.60
	6^+	1340.86	1373.4	-2.43	1404.55	-4.75
	8^+	1959.2	1952.9	0.32	1975.59	-0.84
	10^+	2633.1	2555.0	2.97	2563.67	2.64
	12^+	3314.6	3175.2	4.21	3167.16	4.45
	14^+	3836.7	3811.1	0.67	3787.2	1.29
	16^+	4380.4	4461.3	-1.85	4426.54	-1.05
	18^+	5006.6	5125.1	-2.37	5089.09	-1.65
	20^+	5716.7	5802.2	-1.50	5779.68	-1.10
	22^+	6489.3	6492.6	-0.05	6503.94	-0.23

Table 2 (continued

Nuclei	I^{π}	E_{exp} (keV)	E_{NS3}(keV)	(% error)	$E_{our\ work}$(keV)	(% error)
$^{156}_{68}Er_{88}$	24^+	7315.9	7196.0	1.64	7268.23	0.65
	26^+	8082.2	7918.7	2.02	8079.60	0.03
	28^+	8965.0	8650.0	3.51	8945.76	0.22
	30^+	9864.0	9395.0	4.76	9875.10	-0.11
$^{158}_{68}Er_{90}$	2^+	192.15	189.3	1.64	177.05	7.86
	4^+	527.22	534.2	-1.32	518.46	1.66
	6^+	970.34	977.6	-0.75	970.21	0.01
	8^+	1493.47	1492.1	0.09	1498.64	-0.35
	10^+	2072.53	2063.1	0.46	2081.24	-0.42
	12^+	2680.79	2682.5	-0.06	2701.93	-0.79
	14^+	3374.29	3346.0	0.84	3348.51	0.76
	16^+	4026.1	4051.2	-0.62	4011.12	0.37
	18^+	4679.5	4790.7	-2.38	4681.39	-0.04
	20^+	5327.4	5575.3	-4.65	5351.81	-0.46
	22^+	6026.8	6401.1	-6.21	6015.39	0.19
$^{160}_{68}Er_{92}$	2^+	125.8	125.4	0.32	128.67	-2.28
	4^+	389.9	390.9	-0.26	389.9	0
	6^+	765.7	767.5	-0.24	750.84	1.94
	8^+	1229.3	1230.2	-0.07	1189.13	3.27
	10^+	1761.1	1757.7	0.19	1689.25	4.08
	12^+	2340.4	2332.5	0.34	2240.26	4.28
	14^+	2932.7	2940.2	-0.26	2834.46	3.35
	16^+	3466.5	3567.7	-2.92	3466.5	0
	18^+	4022.0	4207.9	-4.62	4132.79	-2.76
	20^+	4663.0	4852.3	-4.06	4831.09	-3.61
	22^+	5385.0	5495.0	-2.04	5560.22	-3.25
	24^+	6178.0	6131.5	0.75	6319.87	-2.30
	26^+	7032.0	6758.3	3.89	7110.45	-1.12
	28^+	7933.0	7372.9	7.06	7933.0	0
	30^+	8870.0	7973.3	10.11	8789.07	0.91
$^{162}_{68}Er_{94}$	2^+	102.04	101.5	0.53	108.15	-5.99
	4^+	329.62	330.3	-0.21	342.02	-3.76
	6^+	666.68	670.6	-0.59	681.23	-2.18
	8^+	1096.7	1102.1	-0.49	1106.74	-0.92
	10^+	1602.83	1602.5	0.02	1600.19	0.17
	12^+	2165.12	2149.4	0.73	2143.45	1.00
	14^+	2745.72	2722.5	0.85	2718.21	1.00
	16^+	3292.4	3304.6	-0.37	3305.75	-0.41
	18^+	3846.6	3881.7	-0.91	3886.64	-1.04
	20^+	4463.2	4443.4	0.44	4440.63	0.51

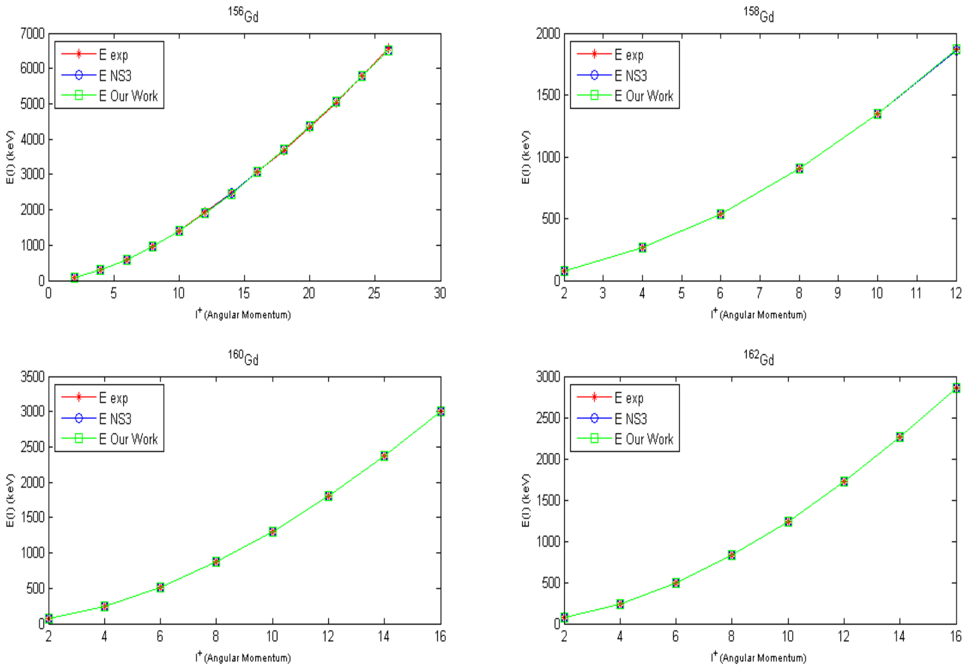

Fig. 1 The calculated NS3, our work and experimental E(I) in (keV) against I of $^{156-162}_{64}Gd$ isotopes

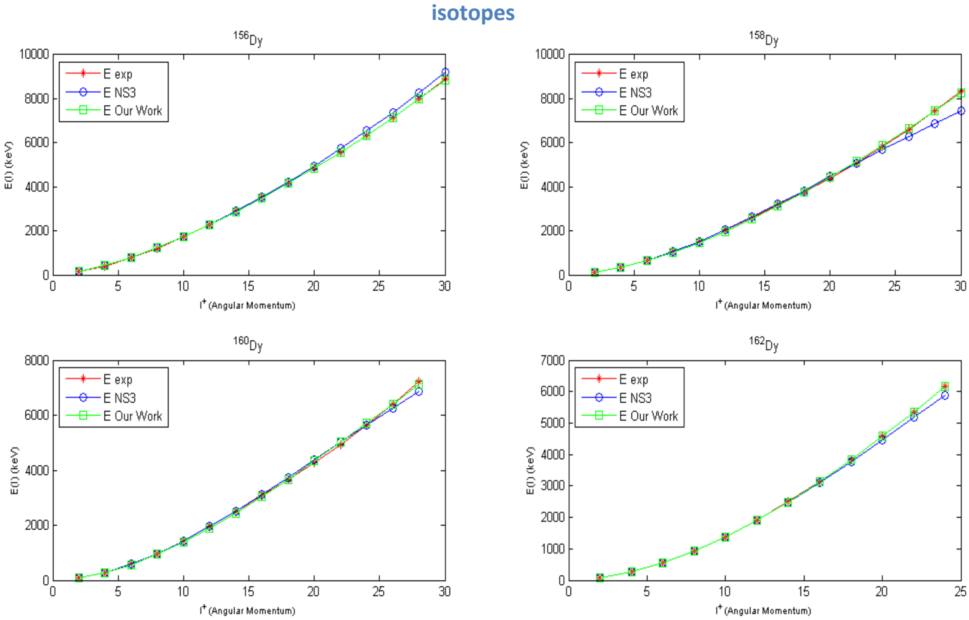

Fig. 2 The calculated NS3, our work and experimental E(I) in (keV) against I of $^{156-162}_{66}Dy$ isotopes

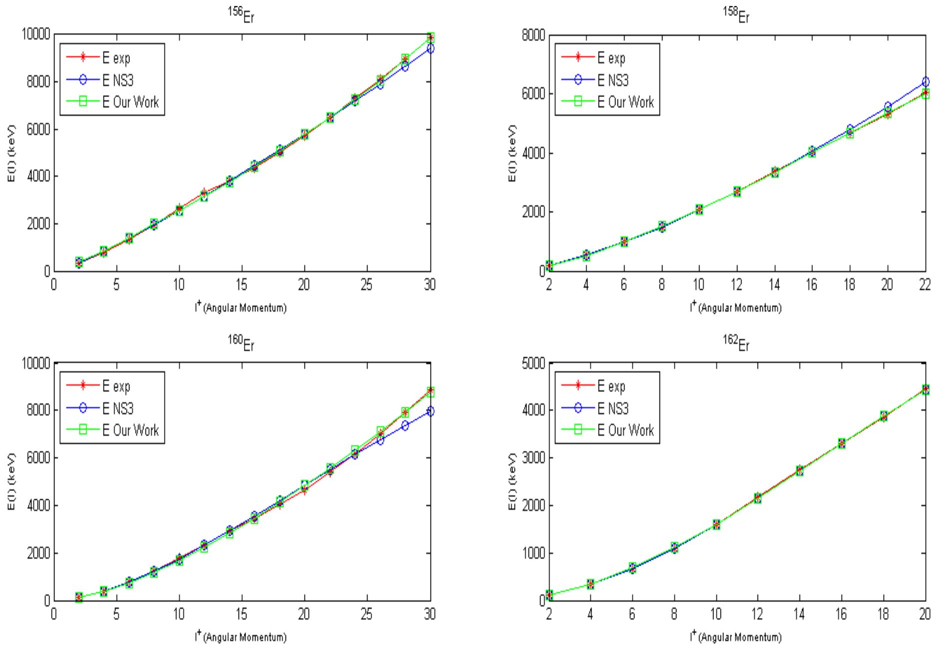

Fig. 3 The calculated NS3, our work and experimental E(I) in (keV) against I of $^{156-162}_{68}Er$ isotopes

Table 3 The experimenal and calculated of E_γ for Gd, Dy and Er even-even isotopes

Nuclei	Transition level $I^\pi \to I^\pi - 2$	E_γ (keV)		
		Experimental	Calculated	
			NS3	Our work
$^{156}_{64}Gd_{92}$	$2^+ \to 0^+$	88.970	90.3	90.46
	$4^+ \to 2^+$	199.22	198.3	197.76
	$6^+ \to 4^+$	296.53	292.9	291.14
	$8^+ \to 6^+$	380.42	375.1	372.47
	$10^+ \to 8^+$	450.94	445.9	443.24
	$12^+ \to 10^+$	508.41	506.6	504.59
	$14^+ \to 12^+$	551.33	558.3	557.42
	$16^+ \to 14^+$	583.68	601.7	602.42
	$18^+ \to 16^+$	613.8	638	640.12
	$20^+ \to 18^+$	652.6	667.7	670.94
	$22^+ \to 20^+$	700.1	691.9	695015
	$24^+ \to 22^+$	752.7	710.8	712.99
	$26^+ \to 24^+$	803.9	725.5	724.6
$^{158}_{64}Gd_{94}$	$2^+ \to 0^+$	79.5143	79.6	79.80
	$4^+ \to 2^+$	181.94	181.7	181.7
	$6^+ \to 4^+$	277.564	277.6	277.27
	$8^+ \to 6^+$	365.098	365.8	365.49
	$10^+ \to 8^+$	445.38	445.0	445.24
	$12^+ \to 10^+$	515.5	514.8	515.49
$^{160}_{64}Gd_{96}$	$2^+ \to 0^+$	75.263	75.3	75.96
	$4^+ \to 2^+$	173.24	173.1	173.45
	$6^+ \to 4^+$	266.31	266.1	265.85
	$8^+ \to 6^+$	353.79	353.5	352.53
	$10^+ \to 8^+$	432.7	433.8	432.88
	$12^+ \to 10^+$	505.6	506.6	506.24

Table 3 (continued)

Nuclei	Transition level $I^{\pi} \rightarrow I^{\pi}-2$	E_{γ} (keV)		
		Experimental	Calculated	
			NS3	Our work
${}^{160}_{64}Gd_{96}$	$14^+ \rightarrow 12^+$	571.0	571.0	571.96
	$16^+ \rightarrow 14^+$	630.8	626.9	629.39
${}^{162}_{64}Gd_{98}$	$2^+ \rightarrow 0^+$	71.6	71.6	71.20
	$4^+ \rightarrow 2^+$	164.8	164.7	164.99
	$6^+ \rightarrow 4^+$	253.6	253.3	253.0
	$8^+ \rightarrow 6^+$	336.2	336.3	335.49
	$10^+ \rightarrow 8^+$	411.7	412.5	411.76
	$12^+ \rightarrow 10^+$	480.7	481.2	481.03
	$14^+ \rightarrow 12^+$	541.6	541.7	542.59
	$16^+ \rightarrow 14^+$	596.9	594.0	595.67
${}^{156}_{66}Dy_{90}$	$2^+ \rightarrow 0^+$	137.77	136.8	138.23
	$4^+ \rightarrow 2^+$	266.42	270.0	274.3
	$6^+ \rightarrow 4^+$	366.25	367.1	372.19
	$8^+ \rightarrow 6^+$	445.17	442.5	445.68
	$10^+ \rightarrow 8^+$	509.41	504.4	503.4
	$12^+ \rightarrow 10^+$	560.86	557.6	550.4
	$14^+ \rightarrow 12^+$	601.94	605.3	590.8
	$16^+ \rightarrow 14^+$	635.48	649.2	626.8
	$18^+ \rightarrow 16^+$	654.89	689.6	660.7
	$20^+ \rightarrow 18^+$	680.9	730.9	693.6
	$22^+ \rightarrow 20^+$	714.0	770.9	727.0
	$24^+ \rightarrow 22^+$	755.7	810.7	761.7
	$26^+ \rightarrow 24^+$	801.6	851.2	798.4
	$28^+ \rightarrow 26^+$	848.2	892.5	837.9
	$30^+ \rightarrow 28^+$	897.4	935.0	880.8
${}^{158}_{66}Dy_{92}$	$2^+ \rightarrow 0^+$	98.918	98.9	102.939
	$4^+ \rightarrow 2^+$	281.221	218.1	217.311
	$6^+ \rightarrow 4^+$	320.573	320.7	310.42
	$8^+ \rightarrow 6^+$	406.168	406.5	387.49
	$10^+ \rightarrow 8^+$	476.12	475.6	452.35
	$12^+ \rightarrow 10^+$	528.8	528.8	507.84
	$14^+ \rightarrow 12^+$	563.4	568.8	556.12
	$16^+ \rightarrow 14^+$	578.1	596.1	598.89
	$18^+ \rightarrow 16^+$	591.0	612.8	637.48
	$20^+ \rightarrow 18^+$	625.8	643.7	672.95
	$22^+ \rightarrow 20^+$	678.1	616.3	706.2
	$24^+ \rightarrow 22^+$	734.7	597.9	737.96
	$26^+ \rightarrow 24^+$	792.6	606.0	768.89
	$28^+ \rightarrow 26^+$	843.7	592.6	799.4
	$30^+ \rightarrow 28^+$	898.0	576.5	830.08
${}^{160}_{66}Dy_{94}$	$2^+ \rightarrow 0^+$	86.7877	86.8	89.93
	$4^+ \rightarrow 2^+$	197.0342	196.9	196.68
	$6^+ \rightarrow 4^+$	297.2441	297.4	289.57
	$8^+ \rightarrow 6^+$	385.762	385.9	370.4
	$10^+ \rightarrow 8^+$	461.06	461.0	440.56
	$12^+ \rightarrow 10^+$	522.28	522.0	501.09
	$14^+ \rightarrow 12^+$	563.19	567.9	552.823
	$16^+ \rightarrow 14^+$	576.04	602.3	596.377
	$18^+ \rightarrow 16^+$	580.25	624.0	632.22
	$20^+ \rightarrow 18^+$	608.74	635.0	660.69
	$22^+ \rightarrow 20^+$	657.21	637.1	682.02
	$24^+ \rightarrow 22^+$	711.7	631.7	696.35
	$26^+ \rightarrow 24^+$	765.2	620.4	703.79

Table 3 (continued)

Nuclei	Transition level $I^{\pi} \rightarrow I^{\pi}-2$	E_{γ} (keV)		
		Experimental	Calculated	
			NS3	Our work
${}^{160}_{66}Dy_{94}$	$28^+ \rightarrow 26^+$	817.8	604.4	704.34
${}^{162}_{66}Dy_{96}$	$2^+ \rightarrow 0^+$	80.661	80.7	82.74
	$4^+ \rightarrow 2^+$	185.003	184.9	186.45
	$6^+ \rightarrow 4^+$	282.86	283.0	282.41
	$8^+ \rightarrow 6^+$	372.76	372.9	370.74
	$10^+ \rightarrow 8^+$	453.8	453.4	451.52
	$12^+ \rightarrow 10^+$	526.02	522.9	524.75
	$14^+ \rightarrow 12^+$	590.55	581.5	590.39
	$16^+ \rightarrow 14^+$	646.9	628.7	648.39
	$18^+ \rightarrow 16^+$	692.38	665.0	698.58
	$20^+ \rightarrow 18^+$	746.4	691.1	740.86
	$22^+ \rightarrow 20^+$	774.67	707.8	775.01
	$24^+ \rightarrow 22^+$	801.2	716.1	800.82
${}^{156}_{68}Er_{88}$	$2^+ \rightarrow 0^+$	344.53	339.3	359.52
	$4^+ \rightarrow 2^+$	452.86	487.6	498.49
	$6^+ \rightarrow 4^+$	543.47	546.5	546.54
	$8^+ \rightarrow 6^+$	618.34	579.5	571.04
	$10^+ \rightarrow 8^+$	673.9	602.1	588.08
	$12^+ \rightarrow 10^+$	681.5	620.2	603.49
	$14^+ \rightarrow 12^+$	522.1	635.9	620.04
	$16^+ \rightarrow 14^+$	543.7	650.2	639.34
	$18^+ \rightarrow 16^+$	626.2	663.8	662.55
	$20^+ \rightarrow 18^+$	710.1	677.1	690.59
	$22^+ \rightarrow 20^+$	772.6	690.4	724.26
	$24^+ \rightarrow 22^+$	826.6	703.4	764.29
	$26^+ \rightarrow 24^+$	766.3	722.7	811.37
	$28^+ \rightarrow 26^+$	882.8	731.3	866.16
	$30^+ \rightarrow 28^+$	899.0	745.0	929.34
${}^{158}_{68}Er_{90}$	$2^+ \rightarrow 0^+$	192.15	189.3	177.05
	$4^+ \rightarrow 2^+$	335.07	344.9	341.41
	$6^+ \rightarrow 4^+$	443.12	443.4	451.75
	$8^+ \rightarrow 6^+$	523.13	514.5	528.43
	$10^+ \rightarrow 8^+$	579.06	571.0	582.6
	$12^+ \rightarrow 10^+$	608.26	619.4	620.69
	$14^+ \rightarrow 12^+$	693.5	663.5	646.58
	$16^+ \rightarrow 14^+$	651.81	705.2	662.61
	$18^+ \rightarrow 16^+$	653.4	739.5	670.27
	$20^+ \rightarrow 18^+$	647.9	784.6	670.42
	$22^+ \rightarrow 20^+$	699.4	825.8	636.58
${}^{160}_{68}Er_{92}$	$2^+ \rightarrow 0^+$	125.8	125.4	128.67
	$4^+ \rightarrow 2^+$	264.1	265.5	261.23
	$6^+ \rightarrow 4^+$	375.8	376.6	360.94
	$8^+ \rightarrow 6^+$	463.6	462.7	438.29
	$10^+ \rightarrow 8^+$	531.8	527.5	500.12
	$12^+ \rightarrow 10^+$	579.3	574.8	551.01
	$14^+ \rightarrow 12^+$	592.3	607.7	594.2
	$16^+ \rightarrow 14^+$	533.8	627.5	632.04
	$18^+ \rightarrow 16^+$	555.5	640.2	666.29
	$20^+ \rightarrow 18^+$	641.0	644.4	698.3
	$22^+ \rightarrow 20^+$	722.0	642.7	729.13
	$24^+ \rightarrow 22^+$	793.0	636.5	759.65
	$26^+ \rightarrow 24^+$	854.0	626.8	790.58
	$28^+ \rightarrow 26^+$	901.0	614.6	822.55

Table 3 (continued)

Nuclei	Transition level $I^{\pi} \rightarrow I^{\pi} - 2$	E_{γ} (keV)		
		Experimental	Calculated	
			NS3	Our work
$^{160}_{68}Er_{92}$	$30^+ \rightarrow 28^+$	937.0	600.4	856.07
$^{162}_{68}Er_{94}$	$2^+ \rightarrow 0^+$	102.04	101.5	108.15
	$4^+ \rightarrow 2^+$	227.58	228.8	233.87
	$6^+ \rightarrow 4^+$	337.06	340.3	339.21
	$8^+ \rightarrow 6^+$	430.02	431.5	425.51
	$10^+ \rightarrow 8^+$	506.13	500.4	493.45
	$12^+ \rightarrow 10^+$	562.29	546.9	543.26
	$14^+ \rightarrow 12^+$	580.6	573.1	574.76
	$16^+ \rightarrow 14^+$	546.68	582.1	587.54
	$18^+ \rightarrow 16^+$	554.2	577.1	580.89
	$20^+ \rightarrow 18^+$	616.6	561.7	553.99

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