Fuzzy Assignment problems

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Abstract: In this paper, we deal with solving a fuzzy Assignment Problem (FAP), in this problem $\tilde{C}$ denotes the cost for assigning the n jobs to the n workers and $\tilde{C}$ has been considered to be triangular fuzzy numbers. The Hungarian method is used for solving FAP by using ranking function for fuzzy costs . A numerical example is considered by incorporating a fuzzy numbers into the costs.

Keywords: Assignment problem, Fuzzy numbers, Robust ranking function, Hungarian method.

1. Introduction

An assignment problem is a special type of linear programming problem which deals with assigning various activities (jobs or task or sources) to an equal number of service facilities (men, machine, laborers, etc.) on one to one basis in such a way so that the total time or total cost involved is minimized and total sale or total profit is maximized or the total satisfaction of the group is maximized.
Lin and Wen (2004), Solved the assignment problem with fuzzy interval number costs by a labeling algorithm. Isabels and Uthra (2012), Presents an assignment problem with fuzzy costs represented by linguistic variables which are replaced by triangular fuzzy numbers. Srinivasan and Getharamani (2013), solving fuzzy assignment problem using one's assignment method and applied Reuben's ranking technique. Saleh (2014), Presents a multi-objective assignment problem with interval parameters. The coefficients of FAP problems are assumed to be exactly known. In practice, the coefficients (some or all) are not exact due to the errors of measurement or vary with market conditions …etc.
In this paper, the proposed the fuzzy assignment problem FAP is solve by the Hungarian method using the ranking function for fuzzy costs. An illustrative example is given for the sake of illustration.

2. Preliminaries

In this section, some basic definitions are presented.

2.1 Fuzzy Sets [4, 8, 12]

Let X denoted a universal set. Then a fuzzy subset $\widetilde{A}$ of X is defined by its membership function

$$\mu_A(x) : X \to [0,1]$$

A fuzzy set $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_A(x)$ and is often written as:

$$\widetilde{A} = \{(x, \mu_A(x)) : x \epsilon X , \mu_A(x) \epsilon [0,1] \}$$

Definition 2.1. ($\alpha - Level\ set$)
The $\alpha - level$ set of a fuzzy set $\widetilde{A}$ is defined as ordinary set A_α for which the degree of its membership function exceed the $Level\ \alpha$:

$$A_\alpha = \{x \in X : \mu_A(x) \geq \alpha , 0 \leq \alpha \leq 1 \} = [A_\alpha^L , A_\alpha^U],$$

Where

$$\begin{cases} A_\alpha^L = inf \ \{x \in X: \mu_A(x) \geq \alpha \} \ and \\ A_\alpha^U = sup \ \{x \in X: \mu_A(x) \geq \alpha \} \end{cases}$$

Definition 2.2. (Normal Fuzzy Set)
A fuzzy set $\widetilde{A}$ is called normalized if $A_1 = \{x \in \mathcal{R} \mid \mu_A(x) = 1\} \neq \varphi$.
The set of all points $x \epsilon X$ with $\mu_A(x) = 1$ is called core of a fuzzy set $\widetilde{A}$.
The support of a fuzzy set $\widetilde{A}$ is the ordinary subset of X

$$supp\ A = \{x \in X:\ \mu_A(x) > 0\}$$

Definition 2.3. (Convex Fuzzy Sets)

A fuzzy set $\widetilde{A}$ in convex set $X = \mathcal{R}^n$ is said to be a convex fuzzy set if and only if its $\alpha - level$ set are convex.

An alternative and more direct definition of a convex fuzzy set as follows:

A fuzzy set $\widetilde{A}$ is convex if and only if

$$\mu_A(\omega x_1 + (1-\omega)x_2) \geq min\{\ \mu_A(x_1), \mu_A(x_2)\ \}$$
$$for\ all\ \ x_1, x_2 \in \mathcal{R}^n\ \ and\ \ \omega \in [0,1].$$

2.2 Fuzzy Numbers [1, 8, 12]

A fuzzy number is a convex normalized fuzzy set of the real line $\mathcal{R}$ whose membership function is piecewise continuous.

Definition 2.4. (Triangular Fuzzy Numbers)

A fuzzy number $\widetilde{A}$ is said to be a triangular fuzzy numbers (T.F.N) if it has the following membership function:

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \dfrac{x-a_3}{a_2-a_3} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

Then we say that $\widetilde{A}$ is triangular fuzzy number, written as:

$$\widetilde{A} = (a_1, a_2, a_3)\ \ where\ \ a_1, a_2, a_3 \in \mathcal{R}$$

The interval of confidence for the triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$ at $\alpha - level\ set$ is defined as:

$$A_\alpha = [\ a_\alpha^L\ ,\ a_\alpha^U] = [a_1 + (a_2 - a_1)\alpha\ ,\ a_3 - (a_3 - a_2)\alpha\ ], \forall\ \alpha \epsilon [0,1]$$

2.2.1 The Arithmetic Operations on Fuzzy Numbers [2, 5, 9]

In this section, we introduce the arithmetic of fuzzy numbers.

Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be two T.F.N, then we will study some operations as follows:

1 **Addition:**

$$\begin{cases} \widetilde{A} + \widetilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) \\ \quad = (a_1 + b_1\ ,\ a_2 + b_2\ ,\ a_3 + b_3) \end{cases}$$

2 **The symmetric (image):** $\{- \widetilde{A} = (-a_3, -a_2, -a_1)$

3 **Subtraction:**

$$\begin{cases} \widetilde{A} - \widetilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) \\ \quad = (a_1 - b_3\ ,\ a_2 - b_2\ ,\ a_3 - b_1), \end{cases}$$

4 **Multiplication:**

$$\widetilde{A} \times \widetilde{B} \cong \begin{cases} (a_1b_1\ ,\ a_2b_2\ ,\ a_3b_3), & a_1 \geq 0 \\ (a_1b_3\ ,\ a_2b_2\ ,\ a_3b_3), & a_1 < 0\ , a_3 \geq 0 \\ (a_1b_3\ ,\ a_2b_2\ ,\ a_3b_1), & a_3 < 0 \end{cases}$$

5 **Division:**

$$if\ \ \ 0 \notin \widetilde{B} = (b_1, b_2, b_3)\ \ \ then\ \ \widetilde{A} \div \widetilde{B} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$$

Remark: As mentioned earlier that $\widetilde{A} - \widetilde{A} \neq \widetilde{0}$, $\frac{\widetilde{A}}{\widetilde{A}} \neq \widetilde{1}$ it is follows that the $\widetilde{C}$ solution of the fuzzy linear equation $\widetilde{A} + \tilde{B} = \widetilde{C}$ is not as would expect $\tilde{B} = \tilde{C} - \widetilde{A}$. The same annoyance appears when solving equation $\widetilde{A} \times \tilde{B} = \widetilde{C}$ whose solution is not given by $\tilde{B} = \frac{\widetilde{C}}{\widetilde{A}}$.

2.3 Ranking function [3, 9]

A ranking function $\Re : F(R) \rightarrow R$ where $F(R)$ set of fuzzy numbers defined on set of real numbers, maps each fuzzy number into real number, where a natural order exists.

2.3.1 Ranking function for triangular fuzzy numbers

The ranking function for $\widetilde{A} = (a_1, a_2, a_3)$ denoted $\Re(\tilde{A})$ proposed by F.Reuben's is defined by :

$$\Re(\tilde{A}) = \frac{1}{2}\int_0^1 (a_\alpha^L + a_\alpha^U)\, d\alpha$$

Where

$$\begin{cases} a_\alpha^L = a_1 + (a_2 - a_1)\alpha \\ , \\ a_\alpha^U = a_3 - (a_3 - a_2)\alpha \quad \forall\, \alpha \epsilon [0,1] \end{cases}$$

Theorm 2.1. If $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers,

$$Then \begin{cases} \widetilde{A} < \widetilde{B} \text{ if and only if } & \Re(\widetilde{A}) < \Re(\widetilde{B}) \\ \widetilde{A} = \widetilde{B} \text{ if and only if } & \Re(\widetilde{A}) = \Re(\widetilde{B}) \\ \widetilde{A} > \widetilde{B} \text{ if and only if } & \Re(\widetilde{A}) > \Re(\widetilde{B}) \end{cases}$$

3. Problem Formulation

3.1 Assignment Problem

The Assignment problem (AP) can be stated in the form of $n \times n$ cost matrix $(C_{ij})_{n\times n}$ of real numbers as given in the table (1) following:

Table.1. Assignment cost

Job → *Person↓*	*Job1*	*Job2*	*Job k*	*Job n*
Person 1	c_{11}	c_{12}	c_{1k}	c_{1n}
Person k	c_{k1}	c_{k2}	c_{kk}	c_{kn}
Person n	c_{n1}	c_{n2}	c_{nk}	c_{nn}

Mathematically assignment problem (AP) can be stated as:

$$(AP): \begin{cases} Min \quad Z = \sum_{i=1}^{i=n}\sum_{j=1}^{j=n} c_{ij}\, x_{ij} \\ Subject\ to: \\ \quad \sum_{i=1}^{i=n} x_{ij} = 1\,, \qquad j = 1,2,\dots,n, \\ \quad \sum_{j=1}^{j=n} x_{ij} = 1\,, \qquad i = 1,2,\dots,n \end{cases}$$

Where $x_{ij} =$

$$\begin{cases} 1, & if\ the\ i^{th}\ person\ assignment\ the\ j^{th}\ job \\ & , \\ 0, & otherwise \end{cases}$$

3.2 Fuzzy Assignment Problems

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only.

2).

Let $\tilde{C}_{ij}$ be the triangular fuzzy numbers cost (payment) if j^{th} job is assignment to p^{th} person (see table The problem is to find an assignment x_{ij} so that the total cost for performing all the jobs is minimum

Table.2. Fuzzy assignment cost				
Job → Person ↓	Job1	Job2	Job k	Job n
Person 1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{1k}$	$\tilde{C}_{1n}$
Person k	$\tilde{C}_{k1}$	$\tilde{C}_{k2}$	$\tilde{C}_{kk}$	$\tilde{C}_{kn}$
Person n	$\tilde{C}_{n1}$	$\tilde{C}_{n2}$	$\tilde{C}_{nk}$	$\tilde{C}_{nn}$

The chosen Fuzzy Assignment Problem (FAP) may be formulated into the following fuzzy linear programming problem:

$$(FAP): \begin{cases} Min\ \tilde{Z} = \sum_{i=1}^{i=n}\sum_{j=1}^{j=n} \tilde{C}_{ij}\, x_{ij} \\ Subject\ to: \\ \quad \sum_{i=1}^{i=n} x_{ij} = 1\,, \quad j = 1,2,\dots,n, \\ \quad \sum_{j=1}^{j=n} x_{ij} = 1\,, \quad i = 1,2,\dots,n \end{cases}$$

Such that $x_{ij} =$

$$\begin{cases} 1, & if\ the\ p^{th}\ person\ assignment\ the\ j^{th}\ job \\ 0\,, & otherwise \end{cases},$$

$\sum_{i=1}^{i=n}\sum_{j=1}^{j=n} \tilde{C}_{ij}\, x_{ij}$: Total fuzzy cost for performing all the jobs.

Definition 3.1. Optimal Solution of Fuzzy Assignment Problems

The optimal solution of the Fuzzy Assignment Problem is the set of non-negative integers $\{x_{ij}\}$ which satisfies the following characteristics:

1. $\sum_{i=1}^{i=n} x_{ij} = 1\,,\ j = 1,2,\dots,n,\quad and$ $\sum_{j=1}^{i=n} x_{ij} = 1\,,\ i = 1,2,\dots,n,$
2. If there exist any set of non-negative integers $\{x^*{}_{ij}\}$ such that $\sum_{i=1}^{i=n} x^*{}_{ij} = 1\,,\ j = 1,2,\dots,n, and\ \sum_{j=1}^{i=n} x^*{}_{ij} = 1\,,\ i = 1,2,\dots,n,$ $then\quad \sum_{i=1}^{i=n}\sum_{j=1}^{j=n} \tilde{C}_{ij}\ x_{ij} \le \sum_{i=1}^{i=n}\sum_{j=1}^{j=n} \tilde{C}_{ij}\ x^*{}_{ij}$

3.3 Assignment Algorithm

An approach is proposed for transformation Fuzzy Assignment Problem (FAP) into a crisp Assignment Problem (AP) using the ranking function for fuzzy assignment cost matrix.

Step 1. For the entire fuzzy assignment cost matrix $(\tilde{C}_{ij})_{n\times n}$, find ranking function for all fuzzy assignment cost matrix $(\Re(\tilde{C}_{ij}))_{n\times n}$

Step 2. Replace the fuzzy cost $\tilde{C}_{ij}$ by their respective $\Re(\tilde{C}_{ij})$, then the fuzzy assignment cost matrix given in the table (3) following as:

Table.3. Ranking Fuzzy assignment cost

Job → Person ↓	Job 1	Job 2	Job k	Job n
Person 1	$\Re(\tilde{C}_{11})$	$\Re(\tilde{C}_{12})$	$\Re(\tilde{C}_{1k})$	$\Re(\tilde{C}_{1n})$
Person k	$\Re(\tilde{C}_{k1})$	$\Re(\tilde{C}_{k1})$	$\Re(\tilde{C}_{kk})$	$\Re(\tilde{C}_{kn})$
Person n	$\Re(\tilde{C}_{n1})$	$\Re(\tilde{C}_{n1})$	$\Re(\tilde{C}_{nk})$	$\Re(\tilde{C}_{nn})$

Mathematically ranking fuzzy assignment problem can be stated as:

$$\begin{cases} Min\ \Re(\tilde{Z}) = \sum_{i=1}^{i=n}\sum_{j=1}^{j=n}\Re(\tilde{C}_{ij})\,x_{ij} \\ \quad Subject\ to: \\ \quad\quad \sum_{i=1}^{i=n} x_{ij} = 1\,, \quad j = 1,2,\dots,n, \\ \quad\quad \sum_{j=1}^{j=n} x_{ij} = 1\,, \quad i = 1,2,\dots,n \end{cases}$$

Where $x_{ij} = \begin{cases} 1, & if\ the\ p^{th}\ person\ assignment\ the\ j^{th}\ job \\ 0, & otherwise \end{cases}$,

Step 3. Since $\Re(\tilde{C}_{ij})$ are crisp values, the above problem is obviously the crisp Assignment Problem **(AP)** which can be solved by Hungarian Method, the optimal solution of the crisp Assignment Problem which the optimal solution of the Fuzzy Assignment Problem.

Numerical Example

Consider a fuzzy assignment problem with rows representing seven persons and columns representing the seven jobs with Assignment cost. The cost matrix $(\tilde{C}_{ij})_{n\times n}$ is given whose elements are triangular fuzzy numbers as follows in the table (4):

Table 4. Fuzzy assignment cost for example

Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7
Person 1	(11,12,13)	(5,6,13)	(7, 9, 11)	(3,10,11)	(3, 8, 13)	(12,14,16)	(7,9,11)
Person 2	(7,9,11)	(9,10,11)	(5,8,11)	(8 ,10, 12)	(5,6,7)	(9,11,13)	(8,10,11)
Person 3	(3,9,15)	(7,8,13)	(6,7,9)	(4,5,12)	(4, 6, 12)	(8,15,18)	(3,4,17)
Person 4	(5,6,13)	(13,17,21)	(5,7,13)	(8, 10, 16)	(8,15,18)	(7,11,15)	(3,9,13)
Person 5	(6,8,10)	(6,9,12)	(9,11,13)	(11,13,15)	(8,9,10)	(6, 8, 12)	(4, 8, 12)
Person 6	(10,14, 18)	(9,13,17)	(4,6,10)	(2,4,18)	(3,8,17)	(6,12,14)	(7,8, 13)
Person 7	(4,7, 10)	(7,8,13)	(5,6,7)	(1,8, 13)	(8,9,14)	(5,12,19)	(7,8,17)

The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

Solution: To solve the problem, we have to determine the ranking index of the cost matrix $(\tilde{C}_{ij})_{n\times n}$. The $\alpha - level$ of fuzzy cost $\tilde{C}_{11} = (11,12,13)$ define as:

$[a_\alpha^L\ , a_\alpha^U] = [11 + \alpha\,, 13 - \alpha]$ *for which*

$\Re(\tilde{C}_{11}) = \frac{1}{2}\int_0^1 (a_\alpha^L + a_\alpha^U)\,d\alpha$

$\Re(\tilde{C}_{11}) = \frac{1}{2}\int_0^1 ((11 + \alpha + 13 - \alpha))\,d\alpha$

$\Re(\tilde{C}_{11}) = 12$

Similarly, the ranking for the fuzzy costs matrix are calculated as follows in the table (5):

Table 5. Ranking fuzzy assignment cost for example							
Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7
Person 1	12	7.5	9	8.5	8	14	9
Person 2	9	10	8	10	6	11	9.75
Person 3	9	9	7.25	6.5	7	14	7
Person 4	7.5	17	8	11	14	11	8.5
Person 5	8	9	11	13	9	8.5	8
Person 6	14	13	6.5	7	9	11	9
Person 7	7	9	6	7.5	10	12	10

Now solving the resulting assignment problem by using Hungarian method:
Let p_i *and* q_j be row i *and* column j minimum costs.

Step 1. For the cost matrix in the table 5, determine the minimum in each column see table 6.

Step 2. For the cost matrix resulting from **Step 1 ,** each column subtracts the column minimum we have the follwos table (7):

Table 6. Min the column							
Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7
Person 1	12	7.5	9	8.5	8	14	9
Person 2	9	10	8	10	6	11	9.75
Person 3	9	9	7.25	6.5	7	14	7
Person 4	7.5	17	8	11	14	11	8.5
Person 5	8	9	11	13	9	8.5	8
Person 6	14	13	6.5	7	9	11	9
Person 7	7	9	6	7.5	10	12	10
Min the column	$q_1 = 7$	$q_2 = 7.5$	$q_3 = 6$	$q_4 = 6.5$	$q_5 = 6$	$q_6 = 8.5$	$q_7 = 7$

Step 3. For the cost matrix resulting from **Step 2,** identify each row minimum and subtract it from all the entries of the row, (see the table 8,9).

Table 7. Subtracts the column minimum							
Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7
Person 1	5	0	3	2	2	5.5	2
Person 2	2	2.5	2	3.5	0	2.5	2.75
Person 3	2	1.5	1.25	0	1	5.5	0
Person 4	0.5	9.5	2	4.5	8	2.5	1.5
Person 5	1	1.5	5	6.5	3	0	1
Person 6	7	5.5	0.5	0.5	3	2.5	2
Person 7	0	1.5	0	1	4	3.5	3

Table 8. Min the row								
Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7	Min the row
Person 1	5	0	3	2	2	5.5	2	$p_1 = 0$
Person 2	2	2.5	2	3.5	0	2.5	2.75	$p_2 = 0$
Person 3	2	1.5	1.25	0	1	5.5	0	$p_3 = 0$
Person 4	0.5	9.5	2	4.5	8	2.5	1.5	$p_4 = 0.5$
Person 5	1	1.5	5	6.5	3	0	1	$p_5 = 0$
Person 6	7	5.5	0.5	0.5	3	2.5	2	$p_6 = 0.5$
Person 7	0	1.5	0	1	4	3.5	3	$p_7 = 0$

Table 9. Subtracts the row minimum							
Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7
Person 1	5	0	3	2	2	5.5	2
Person 2	2	2.5	2	3.5	0	2.5	2.75
Person 3	2	1.5	1.25	0	1	5.5	0
Person 4	0	9	1.5	4	7.5	2	1
Person 5	1	1.5	5	6.5	3	0	1
Person 6	6.5	5	0	0	2.5	2	1.5
Person 7	0	1.5	0	1	4	3.5	3

Step 4. Identify the optimal solution as the feasible assignment associated with the zero elements of the cost matrix obtained in **Step 3,** (see table 10).

Table 10. The optimal solution							
Job → Person↓	Job1	Job 2	Job 3	Job 4	Job 5	Job 6	Job 7
Person 1	5	(0)	3	2	2	5.5	2
Person 2	2	2.5	2	3.5	(0)	2.5	2.75
Person 3	2	1.5	1.25	✗ 0	1	5.5	(0)
Person 4	(0)	9	1.5	4	7.5	2	1
Person 5	1	1.5	5	6.5	3	(0)	1
Person 6	6.5	5	✗ 0	(0)	2.5	2	1.5
Person 7	✗ 0	1.5	(0)	1	4	3.5	3

The optimal solution calls for assignment problem as follows:

person 1→ J_2 , person 2→ J_5 , person 3→ J_7 , person 4→ J_1 , person 5→ J_6 , person 6 → J_4 , person 7 → J_3

The fuzzy optimal total cost is

$$\tilde{Z} = \tilde{C}_{12} + \tilde{C}_{25} + \tilde{C}_{37} + \tilde{C}_{41} + \tilde{C}_{56} + \tilde{C}_{64} + \tilde{C}_{73} = (31\,,40,87\,)$$

4. Conclusion

In this paper, the assignment costs are considered as imprecise numbers described by triangular fuzzy numbers. Moreover, the fuzzy assignment problem has been transformed into crisp assignment problem using ranking function for fuzzy costs matrix and solves it by Hungarian method.

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