Impact of Increasing the Detuning on The Nonlinear Electric Susceptibility in N-type Four Levels atom

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ABSTRACT—
The effect of increasing Rabi frequencies of laser beams in resonance with the frequency of the allowed transitions on the nonlinear electric susceptibility has been studied both with or without considering the detuning in N-type four levels atoms response to the laser beams . The impact of the increasing the Rabi frequencies of laser beams with increasing the detuning of the transitions on the electric susceptibility shows a reduction in the nonlinear electric susceptibility.

Keywords: Rabi frequencies, laser beams, nonlinear electric susceptibility

INTRODUCTION

New generation has been started in the field of Optics since the amazing tool called the Laser was born [1]. One of the too many applications of laser was using it in the field of light-mater interaction [2]. One of the most important results to the laser-mater interaction is the Nonlinear [5-14, 17].The interaction of light with matter has been studied widely, both with quantum treatment [4],or in semiclassical treatment [15].

Nonlinear optics is the branch of optics that describes the behaviour of light in nonlinear media, that is, media in which the polarization P responds nonlinearly to the electric field E of the light [3].

There are several kinds of atoms that interact with laser field . At the resonant absorption gives a very large dispersive optical non-linearity, which can be used to control the propagation of light through the medium [18-20]. One kind of them is the N-type four level atoms [12,16]. There are several studies that made a lot of modifications in the results of the interaction of laser beams with N-type four level atoms [13].Following our work Abuzariba and Mafaa [21] , we are introducing here the impact of the detuning of the transitions on the nonlinear electric susceptibility when three laser beams are interaction N-type four levels atom.

The Theoretical Treatment

To get the interaction of laser beams with N-type four levels atoms we need to use three laser beams with the frequencies ωp , ωc , ωs that in resonance state with the transitions of the levels $|1\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |4\rangle$, respectively [16], as its shown in figure (1).

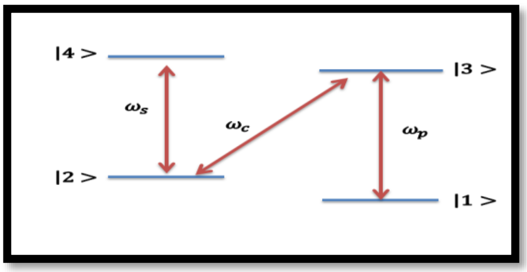

Figure (1):N-type four-level atomic system interacted with three laser fields.

With forbidden transitions $|1\rangle \leftrightarrow |2\rangle$, $|3\rangle \leftrightarrow |4\rangle$, $|1\rangle \leftrightarrow |4\rangle$. The detuning Δ_p generation is shown in figure(2) where the electric field of the laser can be written as

$$\vec{E} = \xi_p \cos(\omega_p t) + \xi_c \cos(\omega_c t) + \xi_s \cos(\omega_s t) + c.c \quad (1)$$

here ξc the amplitude of the coupling laser field that has a frequency ωc , while ξs the amplitude of the second laser field that has a frequency ωs , and ξp the amplitude of the probe laser field that has a frequency ωp

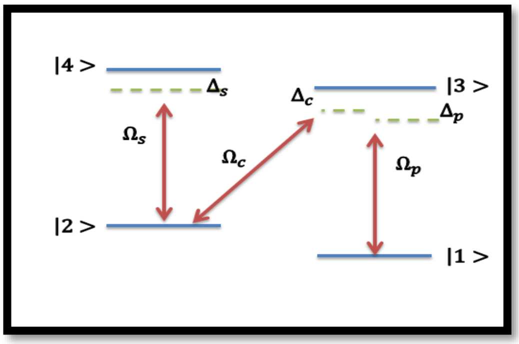

Figure (2): The detuning generation in N-type Four-level atomic system interacted with two laser fields [16].

For this system , the total Hamiltonian is given as

$$H = H_0 + H_I \qquad (2)$$

Were H_0 is the unperturbed Hamiltonian gives with the form

$$H_0 = \begin{bmatrix} \hbar\omega_1 & 0 & 0 & 0 \\ 0 & \hbar\omega_2 & 0 & 0 \\ 0 & 0 & \hbar\omega_3 & 0 \\ 0 & 0 & 0 & \hbar\omega_4 \end{bmatrix} \qquad (3)$$

and the perturbed Hamiltonian is

$$H_I = -E \begin{bmatrix} \hat{p}_{11} & \hat{p}_{12} & \hat{p}_{13} & \hat{p}_{14} \\ \hat{p}_{21} & \hat{p}_{22} & \hat{p}_{23} & \hat{p}_{24} \\ \hat{p}_{31} & \hat{p}_{32} & \hat{p}_{33} & \hat{p}_{34} \\ \hat{p}_{41} & \hat{p}_{42} & \hat{p}_{43} & \hat{p}_{44} \end{bmatrix} \qquad (4)$$

The elements $\hat{p}_{12} = \hat{p}_{21} = \hat{p}_{34} = \hat{p}_{43} = \hat{p}_{14} = \hat{p}_{41} = 0$ since there are forbidden transitions at $|1\rangle \leftrightarrow |2\rangle$, $|3\rangle \leftrightarrow |4\rangle$, $|1\rangle \leftrightarrow |4\rangle$. This gives that the perturbed Hamiltonian can be written as

$$H_I = -E \begin{bmatrix} 0 & 0 & \hat{p}_{13} & 0 \\ 0 & 0 & \hat{p}_{23} & \hat{p}_{24} \\ \hat{p}_{31} & \hat{p}_{32} & 0 & 0 \\ 0 & \hat{p}_{42} & 0 & 0 \end{bmatrix} \qquad (5)$$

Now with the help of rotating wave approximation[22] the final view of the perturbed Hamiltonian can be written as

$$H_I = \frac{-1}{2} \begin{bmatrix} 0 & 0 & \xi_p \hat{p}_{13} e^{i\omega_p t} & 0 \\ 0 & 0 & \xi_c \hat{p}_{23} e^{i\omega_c t} & \xi_s \hat{p}_{24} e^{i\omega_s t} \\ \xi_p \hat{p}_{31} e^{-i\omega_p t} & \xi_c \hat{p}_{32} e^{-i\omega_c t} & 0 & 0 \\ 0 & \xi_s \hat{p}_{42} e^{-i\omega_s t} & 0 & 0 \end{bmatrix} \qquad 6)$$

Where $\Omega_p = \frac{\xi_p |\hat{p}_{13}|}{\hbar}$, $\Omega_c = \frac{\xi_c |\hat{p}_{23}|}{\hbar}$, $\Omega_s = \frac{\xi_s |\hat{p}_{24}|}{\hbar}$ (7)

are the Rabi frequencies of laser beams fields, and the elements of the dipole matrix as

$$\left.\begin{matrix} \hat{p}_{13} = |\hat{p}_{13}| e^{i\varphi_p} \\ \hat{p}_{23} = |\hat{p}_{23}| e^{i\varphi_c} \\ \hat{p}_{24} = |\hat{p}_{24}| e^{i\varphi_s} \end{matrix}\right\} \qquad (8)$$

The total Hamiltonian becomes as

$$H = \frac{\hbar}{2} \times \begin{bmatrix} H_{11} & H_{21} & H_{31} & H_{41} \\ H_{12} & H_{22} & H_{32} & H_{42} \\ H_{13} & H_{23} & H_{33} & H_{43} \\ H_{14} & H_{24} & H_{34} & H_{44} \end{bmatrix} \qquad (9)$$

Where

$H_{11} = 2\omega_1$, $H_{22} = 2\omega_2$, $H_{33} = 2\omega_3$, $H_{44} = 2\omega_4$ (10.a)

$H_{21} = H_{41} = H_{22} = H_{14} = H_{34} = H_{43} = 0$ (10.b)

$H_{13} = -\Omega_p e^{-i\varphi_p} e^{-i\omega_p t}$, $H_{31} = -\Omega_p e^{i\varphi_p} e^{i\omega_p t}$ (10.c)

$H_{23} = -\Omega_c e^{-i\varphi_c} e^{-i\omega_c t}$, $H_{32} = -\Omega_c e^{i\varphi_c} e^{i\omega_c t}$ (10.d)

$H_{24} = -\Omega_s e^{-i\varphi_s} e^{-i\omega_s t}$, $H_{42} = -\Omega_s e^{i\varphi_s} e^{i\omega_s t}$ (10.e)

With simple algebra we can get the newest form of the total Hamiltonian as

$$\tilde{H} = \frac{\hbar}{2} \begin{bmatrix} 0 & 0 & -\Omega_p & 0 \\ 0 & 2\Delta_p & -\Omega_c & -\Omega_s \\ -\Omega_p & -\Omega_c & 2(\Delta_p - \Delta_c) & 0 \\ 0 & -\Omega_s & 0 & 2(\Delta_c - \Delta_s) \end{bmatrix} \qquad 11)$$

Where $\Delta_s = \omega_{42} - \omega_s$, $\Delta_p = \omega_{31} - \omega_p$, $and\ \Delta_c = \omega_{32} - \omega_c$.

By using the density matrix operator and the von Neumann's we can get the elements of the density matrix as

$$\dot{\tilde{\rho}}_{11} = \frac{i\Omega_p}{2}(\tilde{\rho}_{13} - \tilde{\rho}_{31}) + \gamma_{21}(\tilde{\rho}_{22} - \tilde{\rho}_{11}) + \gamma_{41}\tilde{\rho}_{44} + \gamma_{31}\tilde{\rho}_{33} \qquad (12.a)$$

$$\dot{\tilde{\rho}}_{22} = \frac{i\Omega_s}{2}(\tilde{\rho}_{24} - \tilde{\rho}_{42}) - i\Omega_c(\tilde{\rho}_{32} - \tilde{\rho}_{23}) + \gamma_{42}\tilde{\rho}_{44} + \gamma_{32}\tilde{\rho}_{33} \qquad (12.b)$$

$$\dot{\tilde{\rho}}_{33} = \frac{i\Omega_p}{2}(\tilde{\rho}_{13} - \tilde{\rho}_{31}) - \frac{i\Omega_c}{2}(\tilde{\rho}_{23} - \tilde{\rho}_{32}) - \gamma_{43}\tilde{\rho}_{44} + (\gamma_{34} + \gamma_{31} + \gamma_{32})\tilde{\rho}_{33} \qquad (12.c)$$

$$\dot{\tilde{\rho}}_{12} = \dot{\tilde{\rho}}_{21}^* = (-\gamma_{12} - i(\Delta_p - \Delta_c))\tilde{\rho}_{12} - \frac{i\Omega_p}{2}\tilde{\rho}_{32} + \frac{i\Omega_c}{2}\tilde{\rho}_{13} + \frac{i\Omega_s}{2}\tilde{\rho}_{14} \qquad (12.e)$$

$$\dot{\tilde{\rho}}_{13} = \dot{\tilde{\rho}}_{31}^* = (-\gamma_{13} - i\Delta_p)\tilde{\rho}_{13} + \frac{i\Omega_p}{2}(\tilde{\rho}_{33} - \tilde{\rho}_{11}) + \frac{i\Omega_c}{2}\tilde{\rho}_{12} \qquad (12.f)$$

$$\dot{\tilde{\rho}}_{14} = \dot{\tilde{\rho}}_{41}^* = \left(-\gamma_{14} - i\left(\Delta_p - \Delta_c + \Delta_s\right)\right)\tilde{\rho}_{14} - \frac{i\Omega_p}{2}\tilde{\rho}_{34} + \frac{i\Omega_s}{2}\tilde{\rho}_{12} \qquad (12.\mathrm{g})$$

$$\dot{\tilde{\rho}}_{23} = \dot{\tilde{\rho}}_{32}^* = (-\gamma_{23} - i\Delta_c)\tilde{\rho}_{23} - \frac{i\Omega_p}{2}\tilde{\rho}_{21} - \frac{i\Omega_c}{2}(\tilde{\rho}_{33} - \tilde{\rho}_{22}) + \frac{i\Omega_s}{2}\tilde{\rho}_{34} + \frac{i\Omega_p}{2}\tilde{\rho}_{21} \qquad (12.\mathrm{h})$$

$$\dot{\tilde{\rho}}_{24} = \dot{\tilde{\rho}}_{42}^* = (-\gamma_{24} - i\Delta_s)\tilde{\rho}_{24} - \frac{i\Omega_c}{2}\tilde{\rho}_{34} + \frac{i\Omega_s}{2}(\tilde{\rho}_{44} - \tilde{\rho}_{22}) \qquad (12.i)$$

$$\dot{\tilde{\rho}}_{34} = \dot{\tilde{\rho}}_{43}^* = (-\gamma_{34} - i(\Delta_c - i\Delta_s))\tilde{\rho}_{34} - \frac{i\Omega_p}{2}\tilde{\rho}_{14} - \frac{i\Omega_c}{2}\tilde{\rho}_{24} + \frac{i\Omega_s}{2}\tilde{\rho}_{32} \qquad (12.j)$$

and in the study state

$$\dot{\tilde{\rho}}_{12} = \dot{\tilde{\rho}}_{21}^* = (-\gamma_{12} - i(\Delta_p - \Delta_c))\tilde{\rho}_{12} + \frac{i\Omega_c}{2}\tilde{\rho}_{13} + \frac{i\Omega_s}{2}\tilde{\rho}_{14} \qquad (13.a)$$

$$\dot{\tilde{\rho}}_{13} = \dot{\tilde{\rho}}_{31}^* = (-\gamma_{13} - i\Delta_p)\tilde{\rho}_{13} + \frac{i\Omega_p}{2} + \frac{i\Omega_c}{2}\tilde{\rho}_{12} \qquad (13.b)$$

$$\dot{\tilde{\rho}}_{14} = \dot{\tilde{\rho}}_{41}^* = \left(-\gamma_{14} - i\left(\Delta_p - \Delta_c + \Delta_s\right)\right)\tilde{\rho}_{14} + \frac{i\Omega_s}{2}\tilde{\rho}_{12} \qquad (13.\mathrm{c})$$

Simply it is in matrix form $X = MA$ and the polarization operator is

$$P = N(\rho_{13}\hat{p}_{31} + \rho_{31}\hat{p}_{13} + \rho_{23}\hat{p}_{32} + \rho_{24}\hat{p}_{42} + \rho_{32}\hat{p}_{23} + \rho_{42}\hat{p}_{24}) \quad (12)$$

and the real part of the nonlinear electric susceptibility is

$$Rel\left(\chi\right) = \frac{2N|\hat{p}_{13}|}{\varepsilon_0 \xi_p}\frac{\Omega_p}{Z}\{A\left(\Delta_p(\gamma_{12}-\gamma_{14}) + \Delta_c(\gamma_{14}-\gamma_{12}) + \Delta_s\gamma_{42}\right) + B\left(-\gamma_{12}\gamma_{14} + \Delta_p^2 + \Delta_c^2 - \Delta_p(2\Delta_c + \Delta_s) - \Delta_s\Delta_c\right)\} \quad (13.a)$$

$$Img\left(\chi\right) = \frac{2N|\hat{p}_{13}|}{\varepsilon_0 \xi_p}\frac{\Omega_p}{Z}\{A\left(-\gamma_{12}\gamma_{14} + \Delta_p^2 + \Delta_c^2 - \Delta_p(2\Delta_c + \Delta_s) - \Delta_s\Delta_c\right) - B\left(\Delta_p(\gamma_{12}-\gamma_{14}) + \Delta_c(\gamma_{14}-\gamma_{12}) + \Delta_s\gamma_{42}\right)\} \quad (13.b)$$

RESULTS and DISCUSSIONS

With the help of matlap software we did study the effect of increasing the detuning on the nonlinear electric susceptibility. To get a clear view about increasing the detuning in N-type four levels atom interacting with laser beams , we stared with increasing the Rabi frequencies at certain values of the detuning .

The effect of increasing Rabi frequencies on the nonlinear electric susceptibility without considering the detuning. At Rabi frequencies of the transitions $|2\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |4\rangle$ $\Omega_c = \Omega_s =$ 0.3MHz and the detuning $\Delta_c = \Delta_s = \mathbf{0}$. Figure(3) shows the effect of increasing the Rabi frequencies of the transitions $|2\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |4\rangle$ on the both the real and the imaginary parts of the nonlinear electric susceptibility at $\Omega_c = \Omega_s = 0.3$MHz.

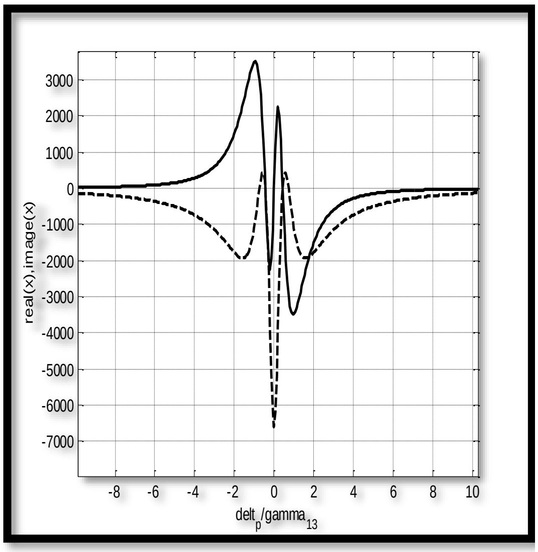

Figure (3):The real (solid) part and the imaginary (dashed) part of the nonlinear electric susceptibility of N-type four levels atom system when $\Omega_c = \Omega_s =$ 0.3MHz $and \Delta_c = \Delta_s = \mathbf{0}\ MHz$.

With increasing Rabi frequency as $\Omega_c = \Omega_s = 1.5MHz$ a bigger values can be noticed in both the absorption and dispersion of the laser energies as it is appearing in figure(4).

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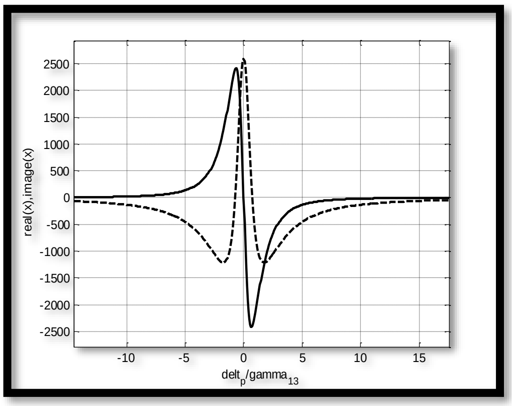

Figure (4):The real (solid) part and the imaginary (dashed) part of the nonlinear electric susceptibility of N-type four levels atom system when $\Omega_c = \Omega_s =$ 1.5MHz $and \Delta_c = \Delta_s = \mathbf{0}\ MHz$

Figure (4) shows an increasing in the absorption (imaginary part of the nonlinear electric susceptibility) of the electric energy, as well as in the dispersion (the real part of the nonlinear electric susceptibility) with both increasing the values of Rabi frequencies of the transitions $|2\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |4\rangle$.

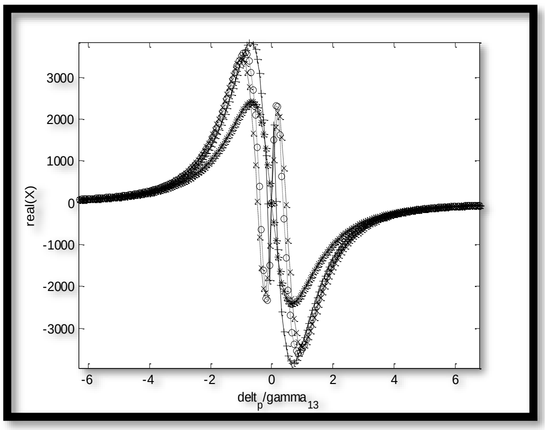

Figure (5): The real part of the nonlinear electric susceptibility of N-type four levels atom system when $\Omega_c = \Omega_s =$ (0.3,0.5,0.8,1.5)MHz and $\Delta_c = \Delta_s = \mathbf{0}\ MHz$.

Figures (5,6) show the real and imaginary parts of nonlinear electric susceptibility, respectively in cases of considering $\Omega_c = \Omega_s =$ (0.3,0.5,0.8,1.5)MHz $and\ \Delta_c = \Delta_s = \mathbf{0}\ MHz$. The decay $\gamma_{13} = 1.5$MHz , whereas the decays $\gamma_{12} = 1.1\text{MHz}, \gamma_{14} = 1.1\text{MHz}$. $\Omega_p = 1.8\text{MHz}$, $\Delta_s = 0$.

1- The effect of increasing Rabi frequencies on the nonlinear electric susceptibility with considering the detuning.

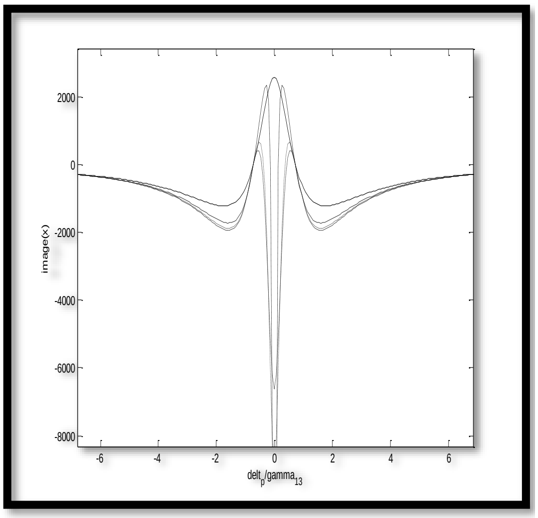

Figure (6): The imaginary part of the nonlinear electric susceptibility of N-type four levels atom system when $\Omega_c = \Omega_s = (0.3, 0.5, 0.8, 1.5)\text{MHz}$ *and* $\Delta_c = \Delta_s = \mathbf{0}\ MHz$.

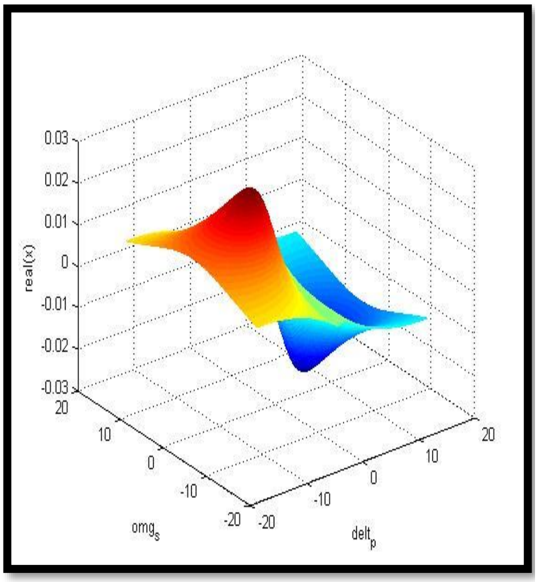

Figure (7):The real part of the nonlinear electric susceptibility of N-type four levels atom system when $\Omega_s = \Delta_p$.

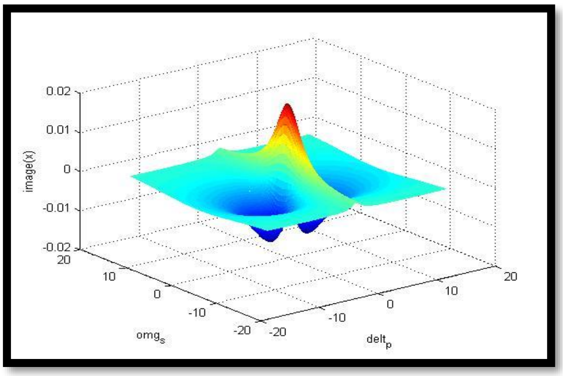

Figure (8) :The imaginary part of the nonlinear electric susceptibility of N-type four levels atom system when $\Omega_s = \Delta_p$.

Figures (7,8) shows both the real and imaginary parts of the nonlinear electric susceptibility as a function of both the detuning of the transition $|1\rangle \leftrightarrow |3\rangle$ Δ_p , the Rabi frequency of the transition $|2\rangle \leftrightarrow |4\rangle$ Ω_s. It is clear that both the the real and imaginary parts of the nonlinear electric susceptibility are decrease with increasing Rabi frequency and the detuning as what we were expected.

CONCLUSIONS

Increasing the Rabi frequencies together with increasing the detuning of the allowed transitions in case of interactions of three laser beams with N-type four levels atom gives a reduction to the nonlinear electric susceptibility. Moreover, The impact of the increasing the Rabi frequencies of the laser beams with increasing the detuning of the transitions on the electric susceptibility has decreases the absorption of the laser beams as it is shown from decreasing the imaginary part of the nonlinear electric susceptibility .

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